Whom Can You Trust? Reputation and Cooperation in Networks

Maia King

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School of Economics and Finance

Queen Mary
University of London

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REPUTATION AND COOPERATION IN NETWORKS

MAIA KING

School of Economics and Finance, Queen Mary, University of London
Blavatnik School of Government, University of Oxford

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Abstract. Community enforcement is an important device for sustaining efficiency in some repeated games of cooperation. We investigate cooperation when information about players' reputations spreads to their future partners through links in a social network that connects them. We find that information supports cooperation by increasing trust between players, and obtain the 'radius of trust': an endogenous network listing the potentially cooperative relationships between pairs of players in a community. We identify two aspects of trust, which relate to the network structure in different ways. Where trust depends on the shadow of punishment, players are trusted if others can communicate about them. This is linked to 2-connectedness of the network and the length of cycles within it. Where trust relates to knowledge of a player's type, players are trusting if they are more likely to receive information through their network connections. Both aspects of trust are linked to new centrality measures that we construct from the probabilities of node-to-node information transmission in networks, for which we provide a novel and simple method of calculation.

JEL classification: C73, D83, D85, L14, Z13.

Keywords: Cooperation, community enforcement, information transmission, networks, imperfect private monitoring, repeated games, reputation, trust.

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1. Introduction

The extent to which individuals can trust and cooperate with each other in the absence of formal enforcement has important effects on economic outcomes and has long been a topic of scholarly interest.[1] Trust consists of "placing valued outcomes at risk of others' malfeasance"(Tilly, 2004) and empirical research on the topic often begins with the survey question: "generally speaking, would you say that most people can be trusted, or that you can't be too careful in dealing with people?" But where does this trust come from? In economic models, there are two main reasons why one person might trust another, even in the face of temptation. Firstly, because their partner could face *punishment* for behaving badly, shifting their *incentives* away from shirking. In this case, if a player knows the expected punishment facing their partner, they can decide whether to trust them on not depending on whether they think the punishment is strong enough to incentivise good behaviour. Secondly, their partner's type could determine their action: they may be a *good type* who is immune to temptation — or a *bad type* who will always cheat no matter what. In this case, if a player knows his partner's type, then he knows whether to trust him or not.

In this paper we build a model which examines both drivers of trust — incentives and types — and show how each aspect depends on the structure of a communication network which connects players. These links could depend on many factors: family and kin relationships; friendships or trading relationships; or proximity given by physical geography such as roads, rivers or the streets of a town. We use a two-sided trust model that allows players to cooperate in a prisoner's dilemma game if and only if they both trust each other. Cooperation is supported by community enforcement, not just retaliation by the victim: other members of the group will punish a deviator if they find out what he has done. [2]

This gives us our main result: a *'cooperation network'* which describes who can cooperate with whom within a community, and which is endogenous to the communication network and the other parameters of the model. This network of cooperation may be quite different from the original communication network, and shows how certain communication network structures can be more or less supportive of overall levels of cooperation, and hence lead to higher (or lower) payoffs.

[1]Coleman (1988); Ostrom (1990); Fukuyama (1996); Putnam et al. (1994); Knack and Keefer (1997); Leeson (2005).

[2]Community enforcement can support cooperation in many settings (Greif, 1993), and its reliance on information transmission between players has been highlighted by Kandori (1992): "*In small communities where members can observe each other's behaviour [...] the crux of the matter is information transmission among the community members.*"

Our model uses Bayesian updating of signals about deviations that travel through the network. In order to do so, we develop a new measure for the probability of node-to-node information transmission in networks. This is based on the *diffusion* approach of Banerjee, Chandrasekhar, Duflo and Jackson (2013), but eliminates the problematic double counting of signals found in their centrality measure, *diffusion centrality*. We also add the possibility that players may choose to conceal signals, but do not allow them to be modified, which means that information is 'evidentiary' (Nava, 2016; Wolitzky, 2014). Using these probabilities of information transmission, we construct two new measures of centrality: *obstructed* and *obstructing centrality*. The more general centrality measure without obstruction is called *word-of-mouth centrality*, which gives the average probability that a signal emitted by each node will be received by any other node in the network.

We find that a pair of players can cooperate if and only if they can both trust each other not to deviate. In particular, a player is *trusted* to cooperate if his position in the network means that other players are able to communicate about him. When trust depends on incentives, what matters is players' expectations of the likelihood of detection and hence punishment. This in turn is linked to *obstructing centrality*: the average probability that other players can communicate about someone, if he tries to obstruct the message.

We also identify a second aspect of trust: a player is *trusting* if his network position means he is likely to detect deviations by others. When trust depends on a knowledge of players' types, payoffs are linked to *obstructed centrality*, which gives the average probability that a player will receive messages that have been obstructed by others. Players who have better knowledge of past play because of their central network position can be more trusting.

We find that both aspects of trust increase with greater probabilities of information transmission, expanding the number of players who can cooperate, and leading to (weakly) higher welfare as information flow increases. These two centrality measures, and hence the two aspects of trust, do not necessarily move together, leading to some surprising results in certain networks. For example, we might expect the centre of a star to be very trusted, because everyone can observe him. Not so. In fact we find that for most parameters in a star network, players on the periphery can cooperate with each other, but the centre is excluded from cooperation. This is because if the centre deviates, the periphery players cannot inform each other, because they are completely dependent upon him for their communication (his obstructing centrality is zero). Hence they cannot trust him not to deviate. We also find cases where the two aspects of trust are diametrically opposed to each other. In a line network, players in the centre of the line cannot be trusted because those at the two ends of the line cannot communicate with each other about the centre's bad behaviour. On the other hand, these central players are very trusting because their network position means they are highly likely to receive signals about others' deviations. In fact, in the line network, those players who are neither in the centre nor the end have the highest capacity to cooperate, echoing the concept of *middle-status conformity* (Phillips and Zuckerman, 2001).[3]

1.1. **Related literature.** The important effect of networks in shaping and influencing economic outcomes has long been emphasised, most recently by Jackson (2008) and Goyal (2007). In particular, there is now a growing literature examining repeated games where interactions and/or monitoring are influenced by a network connecting players, surveyed comprehensively by Nava (2016).[4]

Dixit (2003b) looks at community enforcement when community members are distant from each other. He allows distance to affect three things: the probability players meet, the payoffs if they do, and also the probability they can exchange information. In his model, a continuum of players are arranged around a circle, and cooperation between pairs of players, who are matched in the first period of a two-period game, can be supported by punishment in the final period. A player's incentive to deviate depends on the probability that a true signal emitted by the victim of his deviation will be received by other players: his potential future partners. Dixit finds the 'size of the trading world', an arc of his circle which shows the greatest distance possible between cooperating players, and beyond which they shirk – a similar concept to Fukuyama's (2001) 'radius of trust'. Dixit finds that honesty prevails in a small enough world, and self-enforcing honesty decreases as size increases. He also compares community enforcement to global enforcement with different-sized worlds.

In this paper, we modify Dixit's continuous model of community enforcement in order to apply it to a network setting with discrete players. To do so we need to specify the process by which information is transmitted within the network, using new probabilities of node-to-node information transmission that are based on *diffusion* (Banerjee et al., 2013, 2014), where information flows through a limited number of links, each link with a decay factor that represents the probability that two neighbouring players in the information network pass a signal between them. In our model, the signal denotes a player's 'bad reputation', which Kandori (1992) shows can ensure that a deviator will be punished by other members of the community. Players in our model do not have incentives to lie, and hence we abstract from interesting issues around the possibilities of cheap talk and fabricating rumours, which are studied in detail elsewhere (Ahn and Suominen, 2001; Bloch et al., 2014; Annen, 2011).

[3] I am grateful to Birger Wernerfelt for providing this reference.

[4] Nava and Piccione (2014) examine the case of local public goods, where a player takes the same action with respect to each of his neighbours, while Wolitzky (2013) finds a new centrality measure that can influence a player's robust maximum contribution to global public goods. Karlan et al. (2009), Breza and Chandrasekhar (2015) and Annen (2003) investigate the role of network links in supporting commitment in different settings.

There are many possible ways information can flow in networks (Borgatti, 2005) and many sophisticated information structures have been proposed in this field, and we believe ours is the first model to use probabilistic information flow. Balmaceda and Escobar (2013) and Raub and Weesie (1990) model information as flowing along one link in the network. Renault and Tomala (1998) and Wolitzky (2014) let information flow through all the links in a connected component, finding that the potential for cooperation depends on whether the network is 2-connected. Lippert and Spagnolo (2011) allow information to travel through network links with a delay, and highlight the importance of *gatekeeping* for cooperation, while in an alternative model with delay Kinateder (2008) finds the *diameter* of the network plays an important role. Bloch, Genicot and Ray (2008), Laclau (2014) and Larson (2014, 2017) allow messages to be passed to a subset of players, while in a different setting, Gallo (2014) models information flow in a network as a random walk process.

The communication network in our model means that different players may have different beliefs about past play, depending on the signals they receive from each other through the network, and so our repeated game falls within the class of games of imperfect private monitoring (Kandori, 2002; Sekiguchi, 1997; Bhaskar and Obara, 2002; Chen, 2010). Like Dixit (2003b), our solution concept is perfect Bayesian equilibrium because our players use Bayesian updating when they receive signals through the network. Our probabilistic information flow of signals (not beliefs) allows for Bayesian updating, in contrast to behavioural approaches often used in networks. [5]

In common with much of the literature on cooperation, we model pairwise interactions between players where the stage game is the prisoners' dilemma, as do Lippert and Spagnolo (2011), Ali and Miller (2013), Bloch, Genicot and Ray (2008) and Laclau (2012). In those papers, a network of relationships determines *both* the interaction possibilities and the information flows between players. In contrast, we allow interactions and monitoring relationships to be unrelated to each other, since like Kandori (1992) and Ellison (1994), we have uniform random matching, which is independent across periods. This means that players can play the stage game with partners with whom they do not exchange information, as is the case for Fainmesser (2012) and Fainmesser and Goldberg (2012), although, different to them, our networks are common knowledge. [6]

[5]Degroot (1974); Golub and Jackson (2012). In other settings, Hagenbach and Koessler (2010), Galeotti et al. (2013) and Acemoglu et al. (2014) also focus on transmission of signals in networks, using the cheap talk framework of Crawford and Sobel (1982).

[6]A more general version of the model uses pairwise matching probabilities, defining an *interaction network* that shows which pairs of players are likely to meet and play the stage game. This specification allows us to highlight the importance of two distinct aspects of enforcement: *monitoring* via the information network;

There are two key cooperation-supporting punishment strategies seen most frequently in the literature: contagion, used by Kandori (1992), Ali and Miller (2013) and Jackson, Rodriguez-Barraquer and Tan (2012); and grim trigger or ostracism (sometimes with forgiveness) used by Ahn and Suominen (2001), Raub and Weesie (1990) and Ali and Miller (2016). In common with Dixit (2003b), we apply a different approach: an incomplete information game where players behave cooperatively in order to avoid being mistaken for a *bad type* whom future partners would ostracise. This means that the punishment is renegotiation-proof (Farrell and Maskin, 1989; Benoit and Krishna, 1993; Jackson et al., 2012), and in our setting entails a finitely repeated game (Benoit and Krishna, 1985), in contrast to much of the literature where infinite repetition is used. The bad type also means that this is a game of reputation (Samuelson and Mailath, 2006) and also allows us to pin down expectations off the equilibrium path.

The outline of the paper is as follows. Section 2 describes the network connecting players and the repeated game, and Section 3 describes the equilibrium of interest. Section 4 describes information flow in the network, and Sections 5 and 6 show how levels of trust, cooperation and payoffs depend on the structure of the network. Section 7 concludes the paper.

2. The network and the cooperation game

The repeated game of cooperation has three periods: $t \in \{1, 2, 3\}$. In periods 1 and 3, players are matched in pairs and the stage game is played. In period 2, information is transmitted in the network. There are n players in $N = \{1, ..., n\}$ where $n > 2$ and even.

2.1. **The network.** The n players occupy the nodes of a fixed undirected unweighted network $\mathbf{g}$ such that $\{i, j\} \in \mathbf{g}$ if i and j are neighbours. A walk of length a between two nodes i and j in network $\mathbf{g}$ is a sequence of nodes $(i = x_0, x_1, ..., x_{a-1}, x_a = j)$ such that for every $r \in \{1, 2, ..., a\}$, we have that $\{x_{r-1}, x_r\} \in \mathbf{g}$. If the nodes are distinct, the sequence is a path, and if in addition $i = j$, it is a cycle. Let $\mathbf{G} = [g_{ij}]$ be the adjacency matrix of the network $\mathbf{g}$, where $g_{ij} = 1$ indicates that players i and j are neighbours so $\{i, j\} \in \mathbf{g}$, and $g_{ij} = 0$ otherwise (and $g_{ii} = 0 \ \ \forall i \in N$ by convention). The network $\mathbf{G}$ is common knowledge; all players know each other's network positions. Let $\mathcal{N}_i = \{j : g_{ij} = 1\}$ be the set of player i's neighbours and $|\mathcal{N}_i|$ be i's degree.

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and *sanctioning* via the matching probabilities (Ostrom, 1990; Sobel, 2002). It is available from the author upon request.

As usual, $d_{ij}(\mathbf{G})$ is the length of the shortest path between two players i and j in the network $\mathbf{G}$, which is known as the *social distance*. Let $D_{\mathbf{G}} = \max\{d_{ij}(\mathbf{G})\}$ be the diameter of the network $\mathbf{G}$: the length of the longest shortest path. Two players are *connected* if there exists a path of finite length between them, and a network is connected if all players are connected to each other. If $\mathbf{G}$ is not connected, its diameter is infinite. Let $\mathbf{G}_{-k}$ be the adjacency matrix of the network with player k removed — that is, the $n \times n$ adjacency matrix created when all the entries in the kth row and column of $\mathbf{G}$ are set to zero. If $\mathbf{G}_{-k}$ is connected then the network is *2-connected with respect to* k; the network is *2-connected* if it is 2-connected with respect to all players.

2.2. **The stage game.** The stage game is the prisoners' dilemma with exit (Benoit and Krishna, 1985), augmented by an additional 'dangerous' action B_i, which is very damaging for a player's partner: e.g. 'steal everything'. The action space for each player $i \in N$ is $A_i = \{C_i, D_i, O_i, B_i\}$ where C_i is cooperation and D_i is defection, O_i is exit and B_i is the dangerous action. Table 1 shows how payoffs depend on actions.

	$C_{\mu_i^t}$	D_j	O_j	B_j
C_i	1	$-\beta$	0	$-\beta$
D_i	α	σ	0	$-\beta$
O_i	0	0	0	0
B_i	x_i	x_i	$x_i/2$	x_i

TABLE 1. The stage game between i and j. Payoffs are symmetric, and the entries denote player i's payoffs.

There are two types of player in the set $\Xi = \{\text{S-type, B-type}\}$. Most of the players are strategic or 'S-type'; but there are a few bad apples — 'B-types' — who lurk in the population. This is a game of incomplete information as players do not observe each other's types — as well as imperfect monitoring as described earlier. The bad type is included in the game in order to make punishment renegotiation-proof (Samuelson and Mailath, 2006), and to pin down expectations off the equilibrium path. These bad or 'inept' types are sometimes called *commitment types* because they are committed to a certain action. In our case, we use a simple specification for the payoffs of the bad type.

We assume that $x_i = -x$ for S-types and $x_i = x$ for B-types, and that $x > \beta, \alpha > 1 > \sigma > 0$ and $2 > \alpha - \beta$. This implies that the dangerous action B_i is strictly dominant for a B-type and strictly dominated for the S-type. In turn, these assumptions mean that the stage game between two S-types is the usual prisoners' dilemma with exit, which has two Nash equilibria in pure strategies: mutual defection (D_i, D_j) or mutual exit (O_i, O_j). For a game between an S-type player i and a B-type, the only Nash equilibrium is (O_i, B_j).

We make the following assumption about ϕ, an S-type player's prior belief that another player is a B-type, which ensures that an S-type player will not exit against an unknown player he expects to defect.

Assumption 1: $\sigma(1-\phi) - \beta\phi > 0$

Let μ_i^t denote player i's partner in periods $t \in \{1, 3\}$, and μ^t list the partnerships in each period, with $\mu = (\mu^1, \mu^3)$. Players know the identity of their own partner, but not who anyone else has matched with, as in Kandori (1992). Players match in pairs according to uniform random matching[7] where

$$\Pr\{\mu_i^t = j\} = \frac{1}{n} = \Pr\{\mu_j^t = i\} \quad \forall i, j \in N \text{ and } \forall t \in \{1, 3\} \tag{2.1}$$

2.3. **Reputation and network signals.** A player k gets a bad reputation (Kandori, 1992), $r^k = 1$, if and only if their partner in period 1 received a negative payoff.

$$r^k = 1 \iff U^1_{\mu_k^1} < 0 \tag{2.2}$$

Otherwise, they have $r^k = 0$. So a B-type player will always get a bad reputation, as will an S-type player who defected against a cooperating partner.

$$\Pr[r^k = 1 \mid \{k \text{ is B-type}\}, \quad \mu_k^1 \neq k] \quad = \quad 1 \tag{2.3}$$

$$\Pr[r^k = 1 \mid \{k \text{ is S-type}\}, \quad \mu_k^1 \neq k, \quad D_k, C_{\mu_k^1} \in a^1] \quad = \quad 1 \tag{2.4}$$

In contrast to some reputation models, in our case a player's reputation is not publicly available — only some players hear about it. Let $s_i(k) \in \{\{1\}, \emptyset\}$ be the signal that player i sends about player k, where $s_i(k) = 1$ signifies k's bad reputation.[8] For now, we assume that a player emits a signal about another player if and only if that player has a bad reputation and was his period 1 partner. That is, only the true victim of a deviation will emit a signal. In Section 4, where we discuss information transmission in more detail, we show that this behaviour is incentive compatible. Passing information on or concealing it are both costless.

Assumption 2: $s_i(k) = 1 \iff r^k = 1$ and $\mu_k^1 = i \quad \forall i, k \in N$

[7] Dixit (2003b) has different matching probabilities and payoffs that depend on the network. Different payoffs with different partners which could reflect, for example, complementarities in production with players who have different skills or resources. These complementarities could be greater with players who are at greater social distance. A more general version of the model with these additions is available from the author.

[8] Samuelson and Mailath (2006) show how this separating equilibrium with a bad type is one alternative for reputational games; the other is a pooling equilibrium with a 'good type'. See Appendix C.1 for this alternative in our model. Spence (1973) and Breza and Chandrasekhar (2015) use a specification of the reputation model where both good and bad signals are emitted.

After the signal is emitted, it travels stochasitically through the network and may or may not be received by other players. Let $\rho_j(k) \in \{\{1\}, \emptyset\}$ be the signal that j receives about k. The probability $p_{ij}(k)$ that a signal emitted by i about k will reach j is as follows.

$$\Pr\left[\rho_j(k) = 1 \mid s_i(k)\right] = \begin{cases} p_{ij}(k) & \text{if } s_i(k) = 1 \\ 0 & \text{if } s_i(k) = \emptyset \end{cases} \quad \forall i, j, k \in N \tag{2.5}$$

Due to Assumption 2, note that $p_{ij}(k) = 0$ if $k \in \{i, j\}$, because a player cannot emit a signal about himself — only his victim can.

2.4. **The repeated game.** Let $\rho_j = (\rho_j(k))_{k \in N}$ be player j's 'network signal'. Player i's history (information set) is empty in period 1, and is given by $h_i^3 = (\mu_i^1, r^i, r^{\mu_i^1}, \rho_i)$ at the beginning of period 3. That is, he knows his own and his period 1 partner's reputations, and could also receive a network signal about any other player. But he has not observed his period 1 match's type, only their reputation.

Let $a_i^t \in A_i$ be i's action in the stage game in period $t \in \{1, 3\}$, and let a^t list the actions in period t. Let $b_i = (a_i^1, a_i^3)$ be player i's pure strategy in the repeated game where $b_i \in B_i = \{\{a_i^1, a_i^3\} \mid a_i^1 : \mu_i^1 \to A_i, \quad a_i^3 : \{\mu_i^3, h_i^3\} \to A_i\}$. Let the pure strategy space be $B = \prod_{i \in N} B_i$ and $b = (b_i)_{i \in N}$ be a pure strategy profile of the repeated game.

A player's payoff in each period depends only on his own action and that of his partner, and be given by $U_i(a_i^t, a_{\mu_i^t}^t)$ for periods $t \in \{1, 3\}$. Players are risk-neutral von Neumann-Morgenstern expected utility maximisers, with expected utility function $u_i(\cdot)$. Given players' common discount factor $\delta \in (0, 1]$, payoffs in the repeated game with strategy profile b and realised matches μ are given by

$$u_i(b, \mu) = U_i(a_i^1, a_{\mu_i^1}^1) + \delta U_i(a_i^3, a_{\mu_i^3}^3) \tag{2.6}$$

The repeated game is defined as the tuple:

$$F \equiv (N, (B_i)_{i \in N}, (u_i)_{i \in N}, \Xi, \phi, [p_{ij}(k)]_{i,j,k \in N})$$

2.5. **Timing of the game.** In summary, the order of the game is as follows:

Period 1:

- Players are randomly matched for period 1: μ^1 is chosen
- Players choose actions a^1 and receive payoffs
- Players' reputations are updated, given their partner's payoffs. Each player i observes his own and his partner's reputations $r^i, r^{\mu_i^1}$ with certainty
- For any player i with $r^i = 1$, a signal is emitted by his partner, $s_{\mu_i^1}(i) = 1$

Period 2:

- Information travels between players according to the probabilities $[p_{ij}(k)]_{i,j,k\in N}$

period 3:

- Players observe a network signal ρ_i
- Players are randomly matched for period 3: μ^3 is chosen
- Players choose actions a^3 and receive payoffs

3. Equilibrium

In this section we construct an equilibrium with cooperation; in particular, where cooperation in period 1 can be supported by community enforcement in period 3. [9] This equilibrium has the following structure. In period 1, players either cooperate against a cooperating partner or defect against a defecting partner. In period 3, almost all players defect. Those who do not defect choose exit, which happens if and only if they know that their period 3 partner deviated from the equilibrium strategy in period 1. So if a player deviates by defecting against a cooperating partner in period 1, he knows that any player he is matched with in period 3 — if they find out about his deviation — will choose exit against him, not defection. According to Table 1, mutual defection gives a strictly positive payoff and exit gives a zero payoff to both players, so a player could lose out on positive period 3 payoffs if he deviates in period 1. This expected loss — this punishment — can sustain cooperation.

At this equilibrium, actions may not be symmetric — in period 1 some players may be cooperating while others may not — but all players use the same decision rule for their action choice, a rule that is based on the expected probability of punishment. We are particularly interested in how many players cooperate at this equilibrium, and which ones they are with respect to their network position.

Proposition 3.1. *A perfect Bayesian equilibrium in the repeated game F for all S-type players $i \in N$ is given by the following equilibrium strategy:*

Period 1: *Player i cooperates with his partner j if and only if his expected losses from deviation are above a threshold value, that is $L_i^j \geq L^*$, and this is also the case for his partner so that $L_j^i \geq L^*$. Otherwise, he defects.*

Period 2: *Player i passes on signals about other players $k \neq i$, but does not pass on signals about himself.*

[9] We expect there to be many equilibria of this game; we do not attempt to characterise them here. Like Dixit (2003b), we focus only on our equilibrium of interest.

Period 3: *Player i exits against his partner k if and only if he believes that he is a B-type: either having heard a signal about him or having matched with him in the previous period. Otherwise, he defects.*

Proof. See Propositions 3.2,4.1 and 5.2, and Remark 3.1. ■

If there were no B-types, this equilibrium would be subgame perfect Nash equilibrium (Benoit and Krishna, 1985). The B-types imply that the punishment is renegotiation-proof[10] and also allow us to pin down expectations off of the equilibrium path.[11] The equilibrium concept is perfect Bayesian equilibrium, because players update their beliefs about their partner's type according to the signal(s) they receive and Bayes' rule.

To construct this equilibrium, we proceed by backward induction and examine only the payoffs and decisions of the S-type players. To simplify the notation in this section, let player i's partner in period 1 be player $j = \mu_i^1$, and his partner in period 3 be player $k = \mu_i^3$.

3.1. **Period 3.** In period 3 each player knows his own reputation and that of his period 1 partner, r^i and r^j; and his network signal ρ_i, but he does not know who anyone else matched with or what transpired in those matches. There are two general possibilities for player i's history in period 3. If he met another S-type in period 1, he has history $(r^i = 0, r^j = 0)$ since neither of them deviated. Alternatively i met a B-type in period 1 and has history $(r^i = 0, r^j = 1)$, since he received a negative payoff at the hands of his partner.

In the conjectured equilibrium, if a player hears a signal about another player, he believes him to be a B-type with probability 1. This means that we can combine (2.4) and Assumption 2 to give

$$\Pr[\{k \text{ is B-type}\} \mid \rho_i(k) = 1] = 1 \tag{3.1}$$

Remark 3.1. *The S-type player's equilibrium strategy for period 3 is that he exits if and only if he believes for sure that his partner is a B-type: either having heard a signal about him or having matched with him in the previous period. Otherwise, he defects.*

[10] The equilibrium strategy is not renegotiation-proof when a Pareto-dominating Nash equilibrium of the stage game exists (Farrell and Maskin, 1989; Benoit and Krishna, 1993; Jackson et al., 2012). In our case, players believe a deviator is a B-type, and so the optimal action is exit. If instead, players faced a deviator who was a known S-type, it would be in both players' interests to forgo punishment and switch to the alternative — and Pareto-dominating — Nash equilibrium of mutual defection.

[11] This is because in equilibrium, B-types will deviate and any players who hear about it will update their beliefs on those players' types. Without the B-types, players would not expect anyone to deviate. In this case, if they do hear about a deviation, their beliefs are not clearly specified.

Proof. Clearly, from Table 1, if i believes k is a B-type, his only rational action is to choose exit. If i has not heard a signal, the probability[12] that his partner is a B-type is ϕ. By Assumption 1, this possibility of meeting a B-type is low enough that expected payoffs from choosing defection are positive. ■

Given this equilibrium strategy, we can now identify the payoffs in period 3.

Proposition 3.2. *Let $V_i^j(k)$ be the period 3 expected payoffs in the repeated game F for an S-type player i who met player j, in period 1, did not deviate, and then meets player $k \neq j, i$ in period 3. Period 3 payoffs are*

$$V_i^j(k) \quad = \quad \sigma(1-\phi) - \beta\phi(1 - Q_i^j(k)) \tag{3.2}$$

Where $Q_i^j(k)$ is the conditional probability that an S-type i, having met another S-type j in period 1, hears a signal about a deviation by player k, if k is a B-type.

If i meets the same player $j \neq i$ in both periods 1 and 3, payoffs are $V_i^j(j) = \sigma$ if j was an S-type, and zero if j was a B-type (since i already knows j's type from meeting him in period 1).

Proof. for $V_i^j(k)$ with $k \notin \{i, j\}$, there are two possibilities: either k is a B-type, or he is an S-type. With probability $1-\phi$, k is an S-type, and payoffs are σ as the equilibrium strategy requires both players to defect. With probability ϕ, k is a B-type, and player i's strategy depends on whether or not he has heard a signal about him. Let this probability of a signal being received be given by $Q_i^j(k)$. If i has heard, he will choose exit with payoff 0. If he has not heard, he will choose defection with payoff $-\beta$. ■

Remark 3.2. *The conditional probability $Q_i^j(k)$ is given by $\frac{1}{n-2}\sum_{h \neq i,j} p_{hi}(k)$*

Proposition 3.2 shows how players benefit from being ***trusting***: if there is a higher probability that they receive a signal about a bad reputation, their expected payoffs are increased, because they are more likely to have heard if their partner is a B-type and hence choose exit against him and protect themselves. This is the period of the game where payoffs depend on a player's ability to detect *types.* Here payoffs are increased when a player is more likely to receive signals.

[12]In fact, player i's subjective updated expected probability of each partner k being a B-type could be lower than ϕ, and depends on the network probabilities of information transmission, given in the next Section. See Appendix C.3.

4. Information transmission in the network

In period 2 of our repeated game, information is transmitted through the network. There are many different ways that information can flow in networks (Borgatti, 2005), depending on the nature of the information and the updating approach used by individuals. While Bayesian updating is the standard approach in complete networks, for arbitrary networks the inferences become rather complex, and behavioural approaches are often used.[13] We tackle this complexity by focusing on the transmission of signals, rather than beliefs.[14]

In our model, information flows by *obstructed diffusion*, where a signal passes through each link in the network with a fixed probability p, up to a maximum number of links T. As defined by Banerjee, Chandrasekhar, Duflo and Jackson (2013), the parameter p denotes how likely players are to meet and/or exchange information with their neighbours, and T shows how many rounds of information transmission there are. For example, if $p = 1$ and $T = 1$, information is passed with certainty only to a player's direct neighbours. With Banerjee et al. (2013)'s diffusion process, a player may receive the same signal more than once along different walks in the network, and *diffusion centrality* measures the *expected total number of times* information is transmitted between nodes. As such, the aggregate measure is a sum of probabilities, which could add to more than 1. This implies that it cannot be used as a probability for Bayesian updating, because it suffers from a problem of double counting: the same signal is counted again when it is transmitted along different walks.[15] Banerjee et al. (2013) also use an implicit assumption that that the probabilities that a signal travels along each walk in the network are independent.

This section develops a new closed-form expression for the probabilities of node-to-node information transmission by diffusion. While maintaining the independence assumption, we use De Morgan's laws to remove the problem of double counting. This means that we can describe *whether or not* a signal is received along these different walks, allowing for Bayesian updating in networks.[16] We also show how the probabilities are affected if players choose not to pass on messages, an action which we call obstruction. To do so we assume that information is 'hard' or 'evidentiary' (Nava, 2016; Wolitzky, 2014), in the sense that nodes can choose whether or not to conceal information, but cannot modify it. If they do not pass

[13]Degroot (1974); Golub and Jackson (2012); Mueller-Frank and Neri (2015); Goyal (2016); Levy and Razin (2014).

[14]In other settings, Hagenbach and Koessler (2010), Galeotti et al. (2013) and Acemoglu et al. (2014) also focus on transmission of signals in networks, using the cheap talk framework of Crawford and Sobel (1982).

[15]This problem is related to but distinct from correlation neglect, where players observe the same signal more than once through different walks in the network and erroneously treat each report as a distinct signal.

[16]A comparison of diffusion centrality and our measure is given in Appendix A.

it on, we call this *obstruction*. Passing information on or concealing it are both costless. Let $\mathbf{\Omega}$ denote the information structure of the network such that $\mathbf{\Omega} = \{p, T, \mathbf{G}\}$.

Definition 4.1. ***Obstructed diffusion*** *is a process whereby information flows through the network with adjacency matrix* $\boldsymbol{G}$ *with probability* $p \in (0, 1]$ *along each link, up to a maximum* T *links. Information is evidentiary and the probability of information flowing along each walk in the network is independent.*[17] *The network links, through which a signal can flow by obstructed diffusion, include only those nodes who choose not to obstruct it in that round of information transmission.*

In our model, all signals are true and are distinguished only by their subject — the identity of the player whose bad reputation is being transmitted. The possibility of obstruction provides an action space for the players in period 2 of our cooperation game. There are T rounds (or sub-periods) of information transmission within period 2. Players need to decide whether to pass on signals about each of the players, in each of the rounds of information transmission. This means that the action space for each player in period 2 consists of $\prod_{k \in N, \tau \leq T}$\{pass on signals about player k in round τ, do not pass on signals about player k in round τ\}. Recall from the previous Section that the S-type players' equilibrium strategy in period 3 is \{exit if a signal has been received about your partner, otherwise defect\}. We can now observe the following.

Proposition 4.1. *When players' equilibrium strategies in period 3 are those given in Remark 3.1, and signals flow in period 2 through the network by obstructed diffusion given in Definition 4.1, a player who is the subject of a signal strictly prefers to conceal it, and players who are not the subject of a signal weakly prefer to pass it on. Secondly, the victim of a deviation will truthfully report it, and a player who has not suffered a deviation will not fabricate a report of one.*

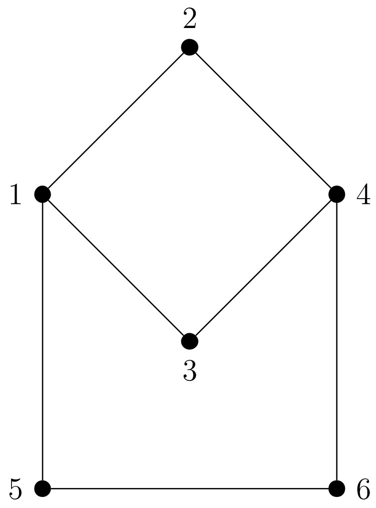
Proof. See Appendix. ■

This means that we can identify the network links through which a signal about a player can travel in period 2 — it is all the network links except those which include the player himself.[18] Next we need to calculate these probabilities given Definition 4.1. To do so, we

[17]This independence is implicitly assumed in Banerjee et al. (2013) and is achieved with the following assumptions: a signal is emitted by the source in each round of information transmission; players pass on each signal they receive independently of whether they receive any other signals; and players do not store information after passing it on to their neighbours, so that the only information players recall after the information transmission process has ended, is that which arrived in the final round of information transmission.

[18]The same approach could be used for more complex obstruction patterns, shown in Appendix C.2.

Figure 1. An example network of information transmission



need a way to account for the fact that a pair of players may be connected to each other by several walks in the network and as such, may transmit a signal via more than one of these walks. We use De Morgan's laws of duality (Fuente, 2000) to tackle this issue. One of these laws states that for a family of sets $\mathbb{A} = \{A_i; i \in I\}$ in the universal set X, where I is some index set, we have that $\sim (\cup_{i\in I} A_i) = \cap_{i \in I}(\sim A_i)$.[19] In other words, the complement of $w_{ij}(\mathbf{\Omega})$ is given by the probability that j does *not* hear a signal from i along *any* of the walks that connect i and j in $\mathbf{G}$.

4.1. **Word-of-mouth probabilities.** Let us begin with the case where there is no obstruction, using the notation w_{ij} for these probabilities. Take for example the network in Figure 1, where we would like to find w_{14}, the probability that a signal emitted by node 1 will be received by node 4. Let us set $T = 2$. There are two walks of length $\leq T$ that a signal could pass from 1 to 4: those via 2 and 3. Each are of length 2, so a signal can pass along each of them with probability p^2. Adding these two probabilities together would give the bilateral[20] entry of diffusion centrality: $d_{14} = p^2 + p^2$. But this does not give the probability that 4 hears a signal from 1, because we need to take account of the fact that the signal could travel along *both* walks. This happens with probability p^4, because the probability of information

[19]With two events and using logic notation, this law can be written as $\neg(A \cup B) = \neg A \cap \neg B$.

[20]Diffusion centrality (Banerjee et al., 2013) is given by $d_i = \left[\sum_{\tau=1}^{T}(p\mathbf{G})^\tau \mathbf{1}\right]_i$. A comparison of diffusion centrality and word-of-mouth centrality (which is constructed from the word-of-mouth probabilities) is given in Appendix A.

flowing along each of these walks is independent. So we find that $w_{14} = p^2 + p^2 - p^4$. We can get this result directly from De Morgan's laws where $1 - w_{14} = (1 - p^2)(1 - p^2)$.

For a more general formula, we know that for each walk of length τ, the probability that the signal does not travel along all the links in that walk by diffusion is $1 - p^\tau$. For a signal *not* to travel from i to j, we need a signal *not* to travel along every possible walk that connects i to j in $\mathbf{G}$, of length $\leq T$.[21]

Definition 4.2. ***Word-of-mouth probability*** *given by* $w_{ij}(\mathbf{\Omega})$ *is the probability that a signal passes from* i *to* j *by diffusion, when there is no obstruction. For any* $\mathbf{\Omega}$*, we have that*

$$w_{ij}(\mathbf{\Omega}) = 1 - \prod_{\tau=1}^{T} [1 - p^\tau]^{l_{ij}(\tau, \boldsymbol{G})}$$

Where $l_{ij}(\tau, \boldsymbol{G}) = [\boldsymbol{G}^\tau]_{ij}$ *is the number of walks between* i *and* j *of length* τ *in the network* $\boldsymbol{G}$*. This result applies* $\forall i \neq j \in N$*, while* $w_{ii}(\mathbf{\Omega}) = 1$*.*

We have assumed that $\mathbf{G}$ is symmetric, which means that ingoing and outgoing probabilities are identical. This could be easily modified for a directed network. Let us say that players can *communicate* if there is a positive probability that a signal sent by one of them will be received by the other. Let us also say that a network is *informative* if every pair of players in the network can communicate, that is $w_{ij}(\mathbf{\Omega}) > 0 \ \ \forall i, j \in N$.

4.1.1. *Independence.* Note that Definition 4.2 only holds if information flows as set out in Definition 4.1; in particular, if the assumption that the probability of a signal travelling along each walk in the network is independent holds. Let us take this opportunity to reflect on the independence assumption. Consider the network in Figure 1, with node 6 removed, and w_{45}, the probability that a signal emitted by node 4 is received by node 5. There are two walks of length 3 between them: $\{4, 2, 1, 5\}$ and $\{4, 3, 1, 5\}$. With the independence assumption, the two walks can be treated independently, so we have that $w_{15} = 1-(1-p^3)(1-p^3) = 2p^3-p^6$.

We can see that node 1 could receive the signal from 4 through two walks, either via nodes 2 or 3. With independence, he would treat these two signals separately. But a more accurate specification might be that there would only be one opportunity for 1 to pass this signal to 5 or not. In this case, $w_{45} = pw_{41} = p(1 - (1 - p^2)(1 - p^2)) = 2p^3 - p^5$. This result can also be found as the probability that the 3 links in each walk are activated separately, minus the probability that all 5 links in both walks are activated. This would be the kind of result generated by Banerjee et al. (2013)'s algorithm of communication centrality. This example

[21] In parallel work, Ambrus, Chandrasekhar and Elliott (2014) use the inclusion-exclusion principle to tackle a similar problem in a different context.

shows that there is a small loss in precision due to the independence assumption. Arguably, this is offset by an easier and quicker computation of the word-of-mouth probabilities as an approximation of this communication mechanism.

4.1.2. *The information structure.* The three aspects of the information structure $\mathbf{\Omega}$ affect the word-of-mouth probabilities as follows.

Proposition 4.2. *There are complementarities between the three aspects of the information structure $\mathbf{\Omega}$ for the word-of-mouth probabilities given in Definition 4.2. In particular, it holds that*

(1) $w_{ij}(\mathbf{\Omega})$ is increasing in p if and only if $\exists\ \tau \leq T$ such that $l_{ij}(\tau, \boldsymbol{G}) \geq 1$
(2) $w_{ij}(\mathbf{\Omega})$ increases as T increases to $T+1$ if and only if $l_{ij}(T+1, \boldsymbol{G}) \geq 1$
(3) $w_{ij}(\mathbf{\Omega})$ increases as a link is added to $\boldsymbol{G}$ if and only if the new link leads to an increase in $l_{ij}(\tau, \boldsymbol{G})$ for any $\tau \leq T$

An increase in p or T or an additional link in $\boldsymbol{G}$ cannot lead to a decrease in any obstructed word-of-mouth probabilities.

Proof. See Appendix. ■

Proposition 4.2 shows that, as would be expected, a network with more links, greater probability of players transmitting messages to their neighbours, and more rounds for information to travel, could have higher probabilities of information transmission.

4.2. **Obstructed word-of-mouth probabilities.** Next we add back in the effect of obstruction:. For our model, we want to find $p_{ij}(k, \mathbf{\Omega})$, the *obstructed word-of-mouth probability* that a signal about player k's deviation, emitted by player i, will be received by player j. From Remark 4.1, we know that a player k, and only k, will obstruct this signal. This means we calculate these probabilities in the same way as the word-of-mouth probabilities, except we use the network $\mathbf{G}_{-k}$ to calculate the number of walks, since player k will not pass on the signal.

Definition 4.3. ***Obstructed word-of-mouth probability*** *given by $p_{ij}(k, \mathbf{\Omega})$ is the probability that a signal passes from i to j by obstructed diffusion given in Definition 4.1, when the signal is obstructed by k. For any $\mathbf{\Omega}$, we have that*

$$\begin{aligned} p_{ij}(k, \mathbf{\Omega}) &= \Pr[\rho_j(k) = 1 \mid s_i(k) = 1] \\ &= 1 - \prod_{\tau=1}^{T} [1 - p^{\tau}]^{l_{ij}(\tau, \boldsymbol{G}_{-k})} \end{aligned}$$

Where $l_{ij}(\tau, \boldsymbol{G}_{-k}) = [\boldsymbol{G}^{\tau}_{-k}]_{ij}$ *is the number of walks between* i *and* j *of length* τ *in the network* $\boldsymbol{G}_{-k}$. *This result applies* $\forall i \neq j, k \in N$, *while* $p_{ii}(k, \boldsymbol{\Omega}) = 1$.

Note that $p_{ij}(k, \boldsymbol{\Omega}) = 0$ if $i = k$ or $j = k$. Proposition 4.2, which describes the effects of the information structure $\boldsymbol{\Omega}$ on word-of-mouth probabilities, also applies to the case with obstruction, if we replace $\mathbf{G}$ with $\mathbf{G}_{-k}$. Since $\mathbf{G}$ has weakly more links than $\mathbf{G}_{-k}$, from Proposition 4.2, we have that $w_{ij}(\boldsymbol{\Omega}) \geq p_{ij}(k, \boldsymbol{\Omega})$ $\forall i, j, k$.

Finally in this section, we can find centrality measures from the obstructed probabilities, as follows.[22] We find that these centrality measures are linked to payoffs.

Definition 4.4. ***Obstructed centrality*** $P_i(\boldsymbol{\Omega})$ *is the average probability a signal emitted by a player will be received by others, averaged over the possible obstruction of that signal by any of the other nodes in the network.*

$$P_i(\boldsymbol{\Omega}) \quad = \quad \frac{1}{(n-1)^2} \sum_{k \neq i} \sum_{j \neq i} p_{ij}(k, \boldsymbol{\Omega})$$

This centrality measure is linked to period 3 payoffs because it shows how a player's network position affects the average probability that they receive signals via the network. It shows, on average, how *trusting* they are — that is, how easily they can receive signals from the network about other players.

Proposition 4.3. *Period 3 expected payoffs are increasing in obstructed centrality* $P_i(\boldsymbol{\Omega})$, *given in Definition 4.4.*

Proof. See Appendix ■

While we have observed that the B-type will not pass on information about himself, a player cannot infer anything about his neighbour's type just because he does not receive any signal from him. This is due to the stochastic nature of information transmission. [23]

Another centrality is measure is players' *obstructed centrality*, which describes the average probability that other players can communicate, given one player obstructs all their signals.

[22]Word-of-mouth centrality (without obstruction) is given in Appendix A, were it is also compared to diffusion centrality.

[23]However, a player can use the structure of the network to update his beliefs about the likelihood of his period-3 partner being a B-type, given that he did not hear a signal about him. This depends on the network links between his partner's possible period 1 matches and himself and is given in Appendix C.3. But these beliefs would not change his behaviour due to Assumption 1: he would only choose exit against a certain B-type.

Definition 4.5. *A node's* ***obstructing centrality*** $O_k(\mathbf{\Omega})$ *is the average probability other players can communicate if that node obstructs the signals.*

$$O_k(\mathbf{\Omega}) = \frac{1}{(n-1)^2}\sum_{i\neq k}\sum_{j\neq i} p_{ij}(k,\mathbf{\Omega})$$

5. The cooperation network

In this section, we complete our characterisation of the equilibrium by examining conditions under which we can expect cooperation in period 1. Consider the case where player i expects his period 1 partner j to cooperate. If i defects, j will get a negative payoff and send a signal about it, and any S-type player who receives that signal will exit if they are matched with i in period 3.[24]

Let $V_i^j = \frac{1}{n-1}\sum_{k\neq i} V_i^j(k)$ and let $V_i = \frac{1}{n-1}\sum_{j\neq i} V_i^j$ be i's ex ante expected payoffs over all possible period 1 partners j. Now we can write player i's overall expected payoffs from either cooperating or defecting when his S-type partner j cooperates.[25]

$$\begin{aligned} v_i(C_i^1, C_j^1) &= 1 + \delta V_i^j & (5.1)\\ v_i(D_i^1, C_j^1) &= \alpha + \frac{\delta}{n-1}\sum_{k\neq i}\left[\sigma(1-\phi)(1-p_{jk}(i)) - \beta\phi(1-Q_i^j(k))\right] & (5.2)\\ &= \alpha + \delta V_i^j - \frac{\delta\sigma(1-\phi)}{n-1}\sum_{k\neq i} p_{jk}(i) & (5.3)\end{aligned}$$

As shown in (5.3), expected losses that i incurs if he defects against his partner j, who he expects to cooperate, are $\delta\sigma(1-\phi)\frac{1}{n-1}\sum_{k\neq i} p_{jk}(i)$, which is broken down as follows.

- The payoff from mutual defection in period 3 is σ (the 'reward' for cooperation);
- He will only be punished if his future partner is an S-type, which happens with probability $(1-\phi)$;

[24]Note that meeting a B-type in period 1 does not impact a player's incentive for cooperation, because he cannot impose a negative payoff on a B-type and hence cannot get a bad reputation from defecting against him when he should have cooperated. Hence we can exclude the possibility of meeting a B-type in period 1 from our study of the equilibrium incentives for cooperation. In this case his payoffs are $\beta+\sum_{k\neq i,j} V_i^j(k)m_{ik}$, which enter the expression for overall payoffs given later in Proposition 5.3.

[25]By Assumption 1, players will not exit against an unknown player in period 1: they either cooperate or defect.

- Punishment occurs if a signal emitted by j about i reaches his potential future partners $k \in N \setminus i$, the probability of which is given by $p_{jk}(i)$. This is weighted by $\frac{1}{n-1}$, the probability of the match;
- This is summed over all $k \neq i$ potential matches.
- Meanwhile if he matches with j again, payoffs are also zero as j knows for sure about his deviation because $p_{jj}(i) = 1$.

Cooperation requires $v_i(C_i^1, C_j^1) \geq v_i(D_i^1, C_j^1)$. We can rearrange (5.1) and (5.3) to find that cooperation for i, when matched with a partner j who he expects to cooperate, requires expected losses L_i^j from deviation to be above a threshold L^*, where

$$L_i^j \equiv \frac{\delta\sigma(1-\phi)}{n-1} \sum_{k \neq i} p_{jk}(i) \geq \frac{\alpha - 1}{\delta} \equiv L^* \tag{5.4}$$

We can say that if $L_i^j \geq L^*$, then player i is ***trusted*** by player j. This is because player i has an incentive such that if he expects player j to cooperate, player i will also cooperate. We can observe that a player's propensity to cooperate is weakly *increasing in his own losses*. Higher expected losses from a deviation are to a player's advantage because they give him an incentive to be honest. If losses are high enough, he is more likely to be trusted, and therefore more likely to take part in cooperation, with higher payoffs.

In particular, i's expected losses from defecting against j are strictly increasing in the probabilities that j can inform other players — i's potential future matches — about a deviation by i. Player i is more willing to cooperate with j, if j is better able to inform other players about i's bad behaviour. This implies that it is better for incentive-based trust if players can talk about each other.[26] The extent to which they can depends on the network structure and in fact, because of obstruction, on the network structure that remains when each node is removed. This is because each player is unable to commit to passing a message about their own deviation, and has to rely on others to do so, incentivising him with their threat of gossip.

Proposition 5.1. *In period 1, whether players are trusted or not is linked to their obstructing centrality given in Definition 4.5.*

Proof. We can rearrange (5.4) to give the following threshold requirement

$$\frac{1}{n-1} \sum_{k \neq i} p_{jk}(i) \geq \frac{(\alpha - 1)}{\delta\sigma(1-\phi)} \tag{5.5}$$

Taking the average expected loss over all potential period 1 partners j,

[26] Larson (2017) finds a similar result in a different setting.

$$\frac{1}{(n-1)^2}\sum_{j\neq i}\sum_{k\neq i} p_{jk}(i) \geq \frac{(\alpha-1)}{\delta\sigma(1-\phi)} \tag{5.6}$$

Which we can observe is obstructing centrality. ■

This shows us that the extent to which a player is *trusted* depends on their *obstructing centrality* — that is, how easily other players can communicate about them, when they cannot commit to pass information on about themselves.

Incentives play the key role in this period of the model. When incentives for honesty matter, a player wants others to be able to communicate about him, encouraging him to cooperate. This is in contrast to period 3, which focuses on types, and hence where a player wants to be able to communicate about others, in order to get information on their types.

5.1. **Welfare and the cooperation network.** We have noted that if a player's expected losses are less than L^*, he would defect. But if one player in a pair is tempted to defect, knowing this, their partner will defect too, even if their losses would otherwise be high enough to deter a deviation. So we need *both* partners in a pair to have high enough expected losses for cooperation to occur: they must both be trusted by each other. Otherwise, they both defect, coordinating on a Nash equilibrium of the one-shot game and both avoiding the bad reputation.[27]

Proposition 5.2. *A pair of players will cooperate in period 1 if and only if the expected losses of both players in the pair are above a threshold level, L^* for player i matched with player j, $\forall i, j \in N$. Otherwise, they will both defect.*

Proof. If $L_i^j \geq L^*$ and i expects j to cooperate, then i will also cooperate due to (5.4). Similarly if $L_j^i \geq L^*$, and j expects i to cooperate, j will also cooperate. In contrast, if $L_i^j \geq L^*$ but i expects j to defect (which he would if $L_j^i < L^*$), then i will also defect because $\sigma > -\beta$. Equally if $L_i^j \geq L^*$ but $L_i^j < L^*$, both players defect. They also both defect if $L_i^j < L^*$ and $L_j^i < L^*$. Hence both players in a pair will cooperate if and only if both players have losses above the threshold. ■

These equilibrium conditions mean that for any parameters of the model, we can find out which players can cooperate with each other, and which ones cannot. We list these cooperative pairs as the *cooperation network*, $\mathbf{G}^c$, which is endogenous to the information network $\mathbf{G}$ and the other parameters. Individual payoffs in period 1 depend on the number

[27]It would be possible to apply this model to a one-sided trust game rather than a two-sided cooperation (prisoners' dilemma) game, and from that build a directed trust network that shows which players would trust each other in the one-sided game.

of cooperative relationships each player has: that is, their degree (number of neighbours) in the cooperation network. Hence welfare depends on the 'size of the trading world' (Dixit, 2003b) — which in our network case refers the number of edges in $\mathbf{G}^c$.

Proposition 5.3. *The cooperation network* $G^c = [g^c_{ij}]_{ij}$ *is given by*

$$g^c_{ij} = g^c_{ji} = \begin{cases} 1, & \textit{if } L^j_i \geq L^* \textit{ and } L^i_j \geq L^* \\ 0, & \textit{otherwise} \end{cases}$$

and overall expected payoffs in the repeated game F *are given by*

$$v_i = \frac{1}{n-1} \sum_{j \neq i} \left[(1-\phi)(g^c_{ij} + (1-g^c_{ij})\sigma) - \phi\beta + \delta V^j_i \right]$$

Proof. This follows from (3.2) and (5.1). ■

We say that a community of N players has *full cooperation* if and only if $L^j_i \geq L^* \forall i, j \in N$. Using this result, we can identify a link between information and cooperation in our model.

Proposition 5.4. *Trust, cooperation and welfare are weakly increasing in the probabilities of information transmission.*

Proof. See Appendix. ■

The observation that more information supports cooperation and welfare is intuitive, and supports Kandori's (1992) assertion that information about a player's reputation can sustain cooperation within a community. It also echoes results from imperfect private monitoring in infinitely repeated games by Sekiguchi (1997) and Bhaskar and Obara (2002) where, providing that monitoring is sufficiently accurate, the symmetric efficient payoff can be approximated. Experimental evidence such as that by Gallo and Yan (2015) also finds that information plays an important role in supporting cooperation.

6. Trust, visibility and obstruction

Having shown that a network which supports greater information transmission can support more cooperation, next we analyse the effect of different network positions on individual levels of trust, cooperation and payoffs. We identified two aspects of trust — players are trusted if other players can send and receive signals about them, whereas they are trusting if they are likely to receive signals about other players. We have linked these aspects of trust to two centrality measures, and next we examine in more detail how network positions are linked with these two aspects of trust.

6.1. **Visibility.** We have shown that if player i emits a signal that is obstructed by player k, the probabilities that the signal is received by other nodes are determined by player i's position in $\mathbf{G}_{-k}$, the network omitting k. We now define the concept of visibility in networks.

Definition 6.1. *A node is **visible** if and only if everyone can still communicate, even when he is obstructing the signal. That is, player k is visible iff $p_{ij}(k, \mathbf{\Omega}) > 0 \;\; \forall\; i, j \in N \setminus k$.*

Visibility depends on what the network looks like when a node is absent i.e. the structure of $\mathbf{G}_{-k}$. It requires two things for k to be visible. First, $\mathbf{G}_{-k}$ must be connected, because a visible player does not disconnect the network by his absence, i.e. if $\mathbf{G}_{-k}$ is connected then $\mathbf{G}$ is 2-connected with respect to k. Secondly, the diameter of $\mathbf{G}_{-k}$ must be not be greater than T, the maximum distance a signal can travel.[28] For example, in a tree network, only the leaf nodes are visible because every other player would disconnect the network by his absence.

Remark 6.1. *It holds that:*

- *For i, j such that $d_{ij}(\boldsymbol{G}_{-k}) \leq T$, we have that $p_{ij}(k, \mathbf{\Omega}) \geq p^{d_{ij}(\boldsymbol{G}_{-k})} > 0$*
- *For i, j such that $d_{ij}(\boldsymbol{G}_{-k}) > T$, we have that $p_{ij}(k, \mathbf{\Omega}) = 0$*

This implies that:

- *If $\boldsymbol{G}_{-k}$ is connected and $D_{\boldsymbol{G}_{-k}} \leq T$, then $p_{ij}(k, \mathbf{\Omega}) \geq p^{D_{\boldsymbol{G}_{-k}}} > 0 \;\; \forall i, j$ and player k is visible.*
- *If the network is 2-connected and $\max_{k \in N}\{D_{\boldsymbol{G}_{-k}}\} \leq T$, all nodes are visible.*
- *If, in addition, $p = 1$, then $p_{ij}(k, \mathbf{\Omega}) = 1 \;\forall\; i, j, k$, and there is perfect information.*
- *If $\boldsymbol{G}_{-k}$ is not connected, then $\forall i \;\exists\; j$ such that $p_{ij}(k, \mathbf{\Omega}) = 0$*

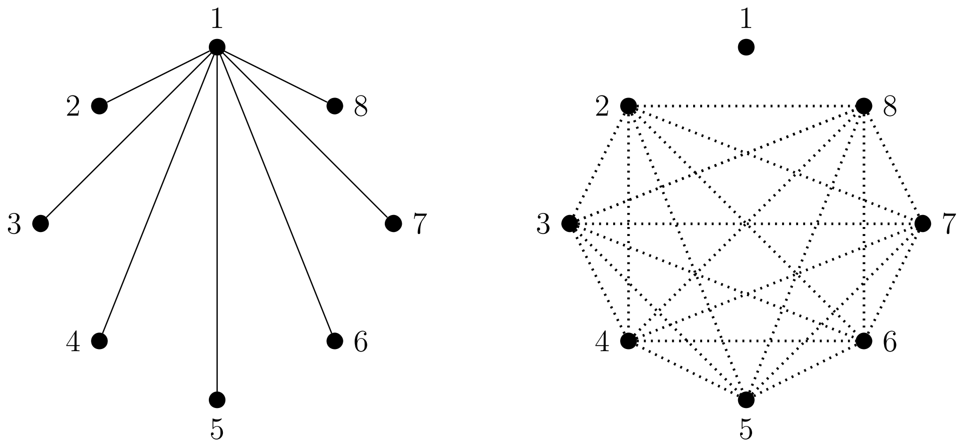
6.2. **Payoffs and visibility.** Only a visible player has a positive probability that everyone could find out if he deviates against any of his matches, so only a visible player can risk the higher losses from a deviation in every match. So a lack of visibility reduces his losses and hence his likelihood of cooperation. In particular,

Remark 6.2. *Given that $L_i^j = \frac{\delta\sigma(1-\phi)}{n-1} \sum_{k \neq i} p_{jk}(i)$, let $P_j(i) = \frac{1}{n-1} \sum_{k \neq i} p_{jk}(i)$ be the expected probability that player j can inform i's future partners about a deviation by i.*

- *Player i is visible $\iff$ $P_j(i) \leq n - 1 \;\forall j$.*

[28] This implies 2-connectedness because a network with a finite diameter is connected.

Figure 2. A star network (L) and its corresponding cooperation network (R)



- *If player i is not visible such that* $D_{\boldsymbol{G}_{-i}} > T$ *but* $\boldsymbol{G}_{-i}$ *is connected, then for* j, k *such that* $d_{jk}(\boldsymbol{G}_{-i}) > T$ *we have that* $P_j(i) < n-1$ *and* $P_k(i) < n-1$.
- *If a player is not visible such that* $\boldsymbol{G}_{-i}$ *is not connected then* $P_j(i) < n-1\ \forall j$.

The link between player i's visibility and the connectedness of $\mathbf{G}_{-i}$ echoes the importance of 2-connectedness for cooperation that is highlighted by Renault and Tomala (1998) and Wolitzky (2014), because 2-connectedness is clearly a necessary condition for visibility. Like Kinateder (2008), we also find that the diameter of the network is important for cooperation, although in our case — because of obstruction — it is the diameter of the network that remains when a player is removed that matters. In fact, a sufficient condition for player i's visibility is that $D_{\mathbf{G}_{-i}} \leq T$.

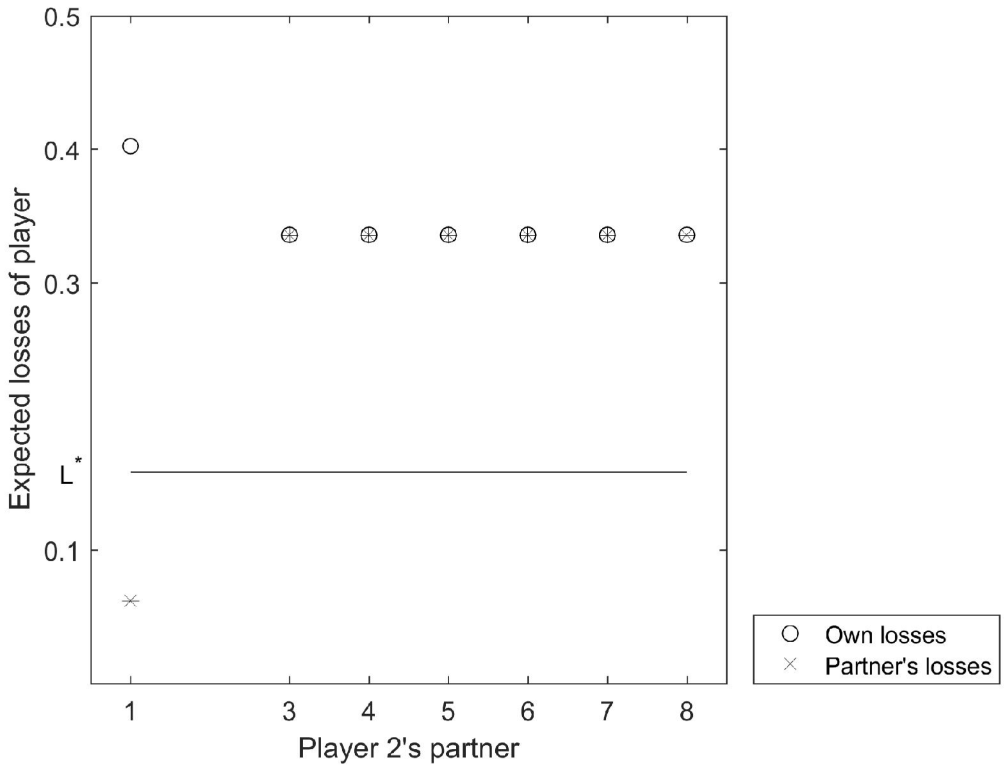
6.3. **Example: star network.** The concept of visibility sheds light on tree networks and in particular the star network.

Remark 6.3. *For a given number of nodes in a connected information network that is informative and a tree, the star network has the most visible nodes of any tree configuration.*

Proof. See Appendix. ∎

We can illustrate the importance of 2-connectedness with an example information network: the star network, shown on the left of Figure 2 with eight players. The star network is not 2-connected with respect to the centre because without him, all other players — the periphery — are singletons. On the other hand, the network is 2-connected with respect

FIGURE 3. Expected losses and threshold losses in star network with eight nodes



to the periphery players because they would not disconnect the network by their absence. Since the star network is not 2-connected with respect to all nodes, it is not 2-connected.

The cooperation network is shown on the right of Figure 2 ($p = 0.5$). Perhaps surprisingly, we observe that the player in the centre of the star cannot cooperate with any other player, while the players on the periphery can all cooperate with each other. To find out why, we can look at the losses in each partnership, shown in Figure 3. The solid line denotes the threshold losses L^* that must be attained by both players in a match to ensure cooperation. The circles show player 2's losses from defecting in each of his partnerships, and the asterisks show his partners' losses.

We can see that player 1, in the centre of the star, can trust player 2 on the periphery. This is because 2's losses from a deviation against 1 are high, since a signal emitted by 1 about 2's deviation only has to travel one link to be received by the other periphery players — 2's potential future partners. But 2 cannot trust 1 in return, because 1's losses when matched with 2 are below the threshold line L^*. This is because if 1 were to deviate against 2, player 1 would obstruct any signal 2 would send about it — and without 1, player 2 is a singleton and so could not tell anyone. So 2 expects 1 to defect and therefore will also defect, and cooperation breaks down between them. This structure recalls the *gatekeeping* and *end network* effects highlighted by Lippert and Spagnolo (2011), because player 1 acts like a 'gatekeeper' of the information network with respect to the periphery players. [29]

On the other hand, as shown in the chart, periphery players have relatively high losses when matched with each other, and these are symmetric, so they can all cooperate with each other. This is because player 1 in the centre of the star provides a walk of length two between all the periphery players, so a signal is very likely to pass between them if any of them deviate, leading to high losses and therefore more trust. The centre of the star misses out on cooperation himself, but supports cooperation by the other players, by ensuring they can communicate with each other.

6.4. **Obstructiveness.** A comparison between word-of-mouth probabilities and obstructed word-of-mouth probabilities gives us a measure of the effect of each player's obstruction.

Definition 6.2. *A node is **obstructive** if and only if his obstruction means that two or more nodes who could previously communicate no longer can. That is, player k is obstructive iff $\exists\, i, j \in N$ such that $w_{ij}(\mathbf{\Omega}) > 0$ and $p_{ij}(k, \mathbf{\Omega}) = 0$.*

Obstruction is linked to social distance, because player k is not obstructive if for all $\{i, j\}$ with $d_{ij}(\mathbf{G}) \leq T$ we also have that $d_{ij}(\mathbf{G}_{-k}) \leq T$. That is to say, player k is not obstructive if he does not increase the social distances too much by his absence from the information network. We collect the conditions for obstructiveness in the following Remark, highlighting the link between obstructiveness and the length of the *cycles* that include a player.

Remark 6.4. *If k has only one neighbour, k is not obstructive. If k has more than one neighbour, then k is not obstructive if and only if the following. For each pair of nodes l, m with $d_{lm}(\boldsymbol{G}) \leq T$ and for whom the sequence (i, k, j) is part of the shortest path(s) between them (implying that $i, j \in \mathcal{N}_k$), we require that $d_{lm}(\boldsymbol{G}) - 2 + d_{ij}(\boldsymbol{G}_{-k}) \leq T$, or equivalently that there exists a cycle in $\boldsymbol{G}$ including the sequence (i, k, j) with length $\leq T + 4 - d_{lm}(\boldsymbol{G})$.*

More generally, if a player k has more than one neighbour, a sufficient condition for him not to be obstructive is if, for each pair of k's neighbours $i, j \in \mathcal{N}_k$, there is a cycle of length ≤ 4 including the sequence (i, k, j). This implies that for all $l, m \in N$ who have the sequence (i, k, j) as part of the shortest path(s) between them, we have that $d_{lm}(\boldsymbol{G}_{-k}) = d_{lm}(\boldsymbol{G})$.

[29] In fact, the periphery nodes could cooperate with the centre if they had additional links to each other. This recalls Myerson's (2008) model of an autocrat whose support depends on the ability of his 'courtiers' to observe his behaviour towards each of them, ensuring fairness.

FIGURE 4. Player k cannot be obstructive in a cycle of length 4 (L), but can be in a cycle length of 5 (R)

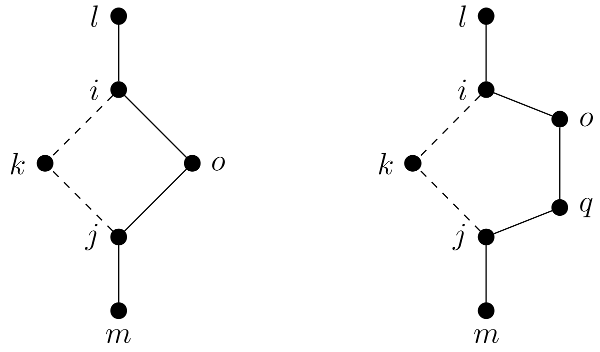


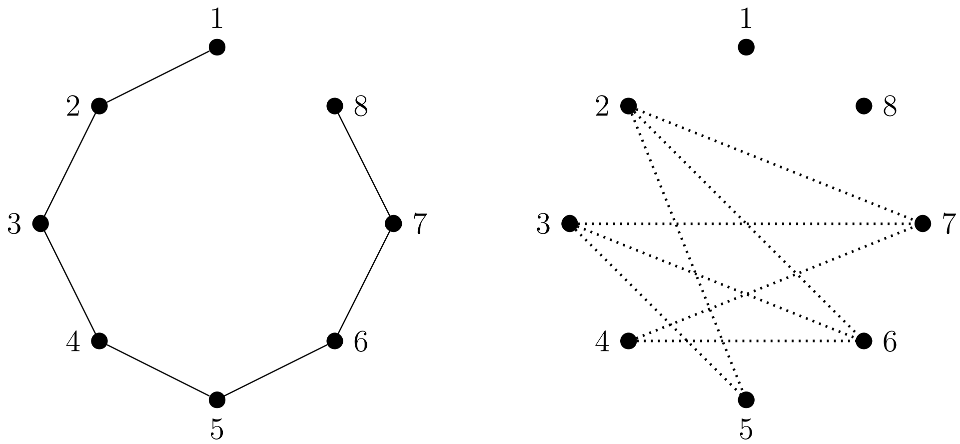
Figure 4 shows an example of this result. In both networks in the Figure, $\mathbf{G}$ is made up of all the solid and dashed links, while $\mathbf{G}_{-k}$ includes only the solid links. In the network on the left, k is in a cycle of four in network $\mathbf{G}$, and $d_{lm}(\mathbf{G}) = 4$. If node k is removed, and we examine $\mathbf{G}_{-k}$, then the social distance between l and m is unchanged — it remains 4. This is because there is an alternative route between l and m via node o. Node o is connected to k's neighbours i and j, so any other walks in a wider network which include the sequence $\{i, k, j\}$ could instead include the sequence $\{i, o, j\}$, which is the same length. So if k is in a cycle of length 4, his removal from the network does not increase the social distances of any other pairs of nodes. So k cannot be obstructive, no matter what the value of T is. On the other hand, the network on the right shows a case where k could be obstructive, depending on the value of T. Now in $\mathbf{G}_{-k}$, the social distance between l and m has increased to 5 links, whereas before when k was present it was only 4. So if $T = 4$, with k passing on signals in the network it would be possible for l and m to communicate. Without him, they cannot: k is obstructive.

The length of the cycle determines whether or not k's neighbours can still communicate, even when k is obstructing those signals. The importance of cycles of length four, our sufficient condition for a player not to be obstructive, recalls well-known results on the importance of network cycles of length three: Coleman's (1988) *closure*; and Jackson et al.'s (2012) *support*.

Proposition 6.1. *Player k is visible if and only if the network is informative ($w_{ij}(\mathbf{\Omega}) > 0 \ \ \forall i, j \in N$) and player k is not obstructive.*

Proof. See Appendix. ■

Figure 5. A line network (L) and its corresponding cooperation network (R)

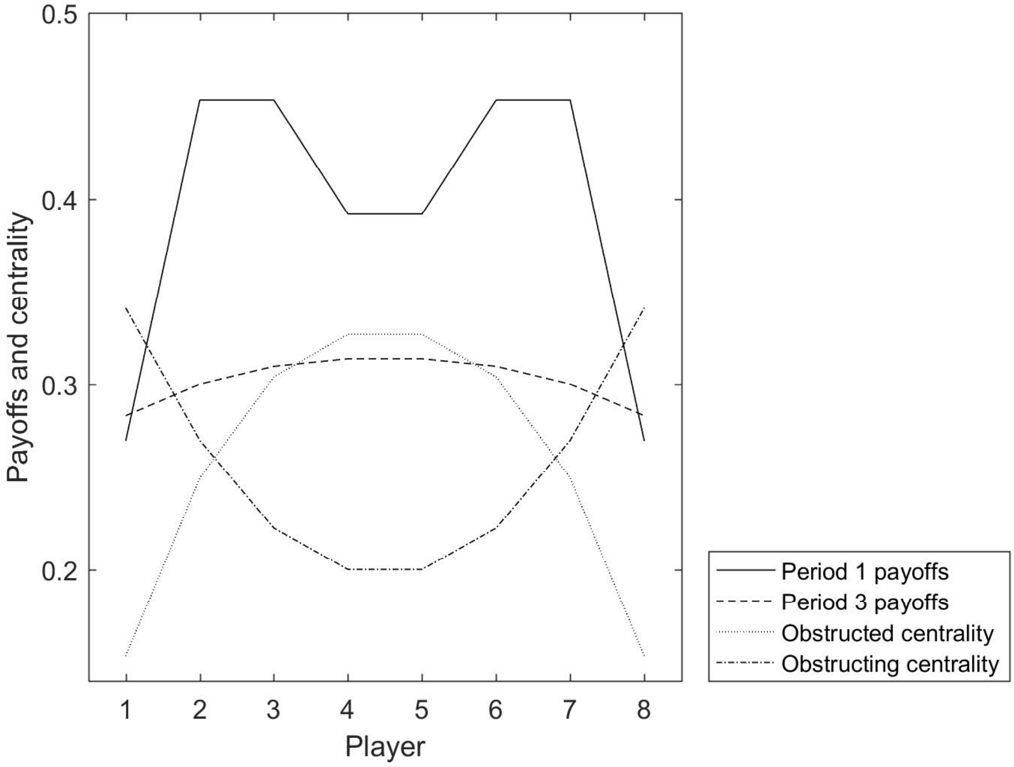


Proposition 6.1 shows that there are two possible reasons why a player may not be visible: firstly, if the network is not informative, so that even without obstruction some nodes cannot communicate; and secondly, if the network is informative but a player is obstructive, in that he can prevent some nodes from communicating if he does not pass signals.

6.5. **Example: line network.** We can illustrate the links between payoffs and the different centrality measures using the line network of eight players, shown on the left of Figure 5. The cooperation network with uniform random matching is shown on the right, and payoffs and obstructed centrality are shown in Figure 6. Obstructed centrality increases for those players located nearer to the centre of the line. As expected from Proposition 4.3, period 3 payoffs rise monotonically with obstructed centrality, because players who are more likely to receive information from the network are more trusting.

On the other hand, there is a non-monotonic relationship between *period 1 payoffs* and obstructed centrality. The chart in Figure 6 shows that players 2, 3, 6 and 7 have the highest cooperation levels in equilibrium, but only moderate levels of obstructed centrality. Looking at the cooperation network, we can see the same pattern: players 1 and 8 are not able to cooperate with anyone, and players 4 and 5 have fewer links in the cooperation network than players 2, 3, 6 and 7. This non-monotonic relationship between centrality ranking and cooperation is similar to a concept known as *middle-status conformity*, identified by sociologists Phillips and Zuckerman (2001), where those with a 'middle' level of status or ranking are most likely to conform to society's norms. In another setting, Butler et al. (2009) also find a non-monotonic relationship between payoffs and trust.

Figure 6. Payoffs and centrality in a line network with eight nodes



6.5.1. *A counterfactual without obstruction.* We can use word-of-mouth probabilities — without obstruction — as a useful counterfactual to investigate the effect of obstruction on cooperation in this network. These are given in Definition 4.2 and allow us to construct counterfactual expected losses given in (5.4) without obstruction. These gives us a counterfactual cooperation network, showing which players would cooperate, were it not for obstruction. This hypothetical network (not shown) includes additional cooperative links between players: 4 with 2, 3 and 5; and 5 with 4, 6 and 7.

This means that the losses without obstruction for the partners of players 1 and 8 are still too low to deter defection; we say that 1 and 8 have *poor network positions* in an *absolute* sense because they are too 'tempting' for anyone to cooperate with, even if they did not obstruct their signals: they can trust no-one. On the other hand, we find that 4 and 5 have three more cooperative links in the counterfactual network; in fact the non-monotonic relationship between cooperation and obstructed centrality disappears when we remove the effect of obstruction. We say that 4 and 5 have *poor relative network positions* because they would have cooperated, were it not for their obstruction of signals. Players with poor network positions in either sense reduce cooperation levels and hence welfare.

This also gives us an insight into the non-monotonic pattern of cooperation shown above. The temptation to cheat *against* poorly-connected players is high. The temptation to cheat *by* well-connected — but obstructive — players is also high. Partnerships that involve these two types of players are less likely to support cooperation because the temptation is to cheat is too great on one or both sides. Pairs of players whose network positions are neither too isolated nor too obstructive have the greatest incentives for honesty.

7. Conclusion

This paper investigates the extent of cooperation in a finitely repeated game in a network setting. We apply Dixit (2003b)'s continuous model to a network: a discrete community of players who occupy its nodes. The players are randomly matched in pairs in the first and last periods, and play the stage game of a modified prisoners' dilemma. From the fixed information network, the model allows us to generate an endogenous network of potentially cooperative relationships. From this we can characterise how levels of cooperation depend on the structure of the information network. Individual players' payoffs are linked to whether their network positions mean they are *trusting* and/or *trusted.* Players are trusting if they are likely to receive information from the network, while they are trusted if others can pass signals about them. A pair of players can only cooperate if they are both trusted by each other. We find that cooperation and welfare both increase with more information. To complete the model which is based on Bayesian updating, we develop a new, simple method for finding the probabilities of node-to-node information transmission in networks, which eliminates the problem of double counting.

We find that players with higher obstructed centrality (constructed from the probabilities of information transmission) receive more information from the network and hence are more trusting. But there can be a non-monotonic relationship between centrality and the extent to which players are trusted, leading to cooperation patterns with middle-status conformity. This is interesting because one might expect the most central player to have the highest payoffs, while we find that a player's central position may actually reduce his capacity to be trusted by others. This is because players cannot commit to pass on a signal about their own bad reputation. Knowing this, players who rely on a central player for information transmission may not trust him, because they know he will obstruct any signal that they send to warn others about his bad reputation. This highlights the importance of 2-connectedness and cycles for cooperation and welfare, because these structures can prevent players from completely obstructing signals about their reputations travelling between other players, ensuring that they are *visible*. Since the non-monotonic relationship between cooperation and centrality disappears in our counterfactual example of a line network without obstruction, we conjecture (though we have no formal proof) that general results may exist linking middle-status conformity to line networks, or other acyclic networks.

The possible link between middle-status conformity and acyclic networks may also be of empirical interest. There is some experimental evidence that players with high centrality may be less 'reciprocal' in trust games (Riyanto and Yeo, 2014; Barr et al., 2009). Obstruction may also imply that acyclic networks are less likely to be observed in communities that use this kind of community enforcement mechanism. Where acyclic networks are present, we may find that central individuals seek other ways to dampen the negative effect of obstruction on their capacity to cooperate. For example, they may enlist their own neighbours (not just the neighbours of their potential victim) as *witnesses* to observe their actions, increasing their potential losses and making them more trustworthy. Secondly, obstructive, bridging players may *specialise* in information transmission: even though they cannot pass signals about themselves and are hence not trusted, they could share in the benefits of cooperation if transfers from other cooperating parties can be arranged. Finally, *local matching* may have a mitigating effect because if someone is more likely to meet the same player again, he will have higher losses from deviating against them, even if that player cannot communicate with others due to obstruction.

Some interesting extensions suggest themselves. The model focusses on the case of a bad reputation, and it may be interesting to fully characterise the case with a 'good label' set out in Appendix C.1, or both good and bad signals, as in Spence (1973) and Breza and Chandrasekhar (2015). It could also be interesting to introduce some stochasticity in order to investigate the effect of risk on cooperative relationships, as observed by Baker (1984). Lastly, it may be possible to use the model to investigate the interaction of formal and informal enforcement regimes, as examined by Kranton (1996), Dhillon and Rigolini (2011) and Dixit (2003a).

Appendix A. Word-of-mouth centrality

In this section we define the probabilities without obstruction, and show how we can construct a centrality measures using these probabilities. We use the term *word-of-mouth* for our non-obstructed probabilities of information transmission between nodes, capturing the intuitive concept whereby information travels within a community via conversations between players and their connections (Ahn and Suominen, 2001; Lippert and Spagnolo, 2011). When there is no obstruction, let $s_i \in \{\{1\}, \emptyset\}$ be the signal emitted by node i, and $\rho_j \in \{\{1\}, \emptyset\}$ be the signal received by node j.

Definition A.1. ***Word-of-mouth probability*** *$w_{ij}(\boldsymbol{\Omega})$ is the probability that a signal emitted by i will reach j by diffusion, given in Definition 4.1, for the case without obstruction.*

$$\Pr\left[\rho_j = 1 \mid s_i\right] = \begin{cases} w_{ij}(\boldsymbol{\Omega}) & \text{if } s_i = 1 \\ 0 & \text{if } s_i = \emptyset \end{cases} \qquad \forall i, j \in N$$

For any $\boldsymbol{\Omega}$, that is, for any $p \in (0,1]$, any T and any $\boldsymbol{G}$, and where $l_{ij}(\tau, \boldsymbol{G}) = [\boldsymbol{G}^\tau]_{ij}$, we have that, $\forall i \neq j \in N$ ($w_{ii}(\boldsymbol{\Omega}) = 1$)

$$w_{ij}(\boldsymbol{\Omega}) = 1 - \prod_{\tau=1}^{T} \left[1 - p^\tau\right]^{l_{ij}(\tau, \boldsymbol{G})}$$

Word-of-mouth centrality *is the average probability of information transmission by diffusion for each player in a network and is given by*

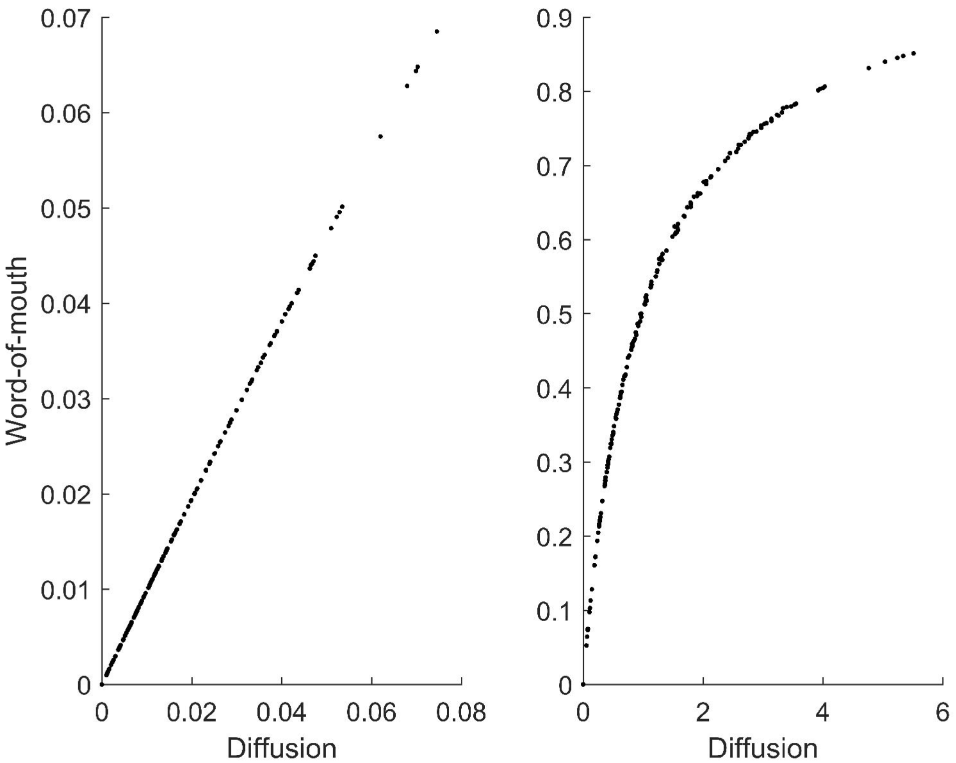
$$w_i(\boldsymbol{\Omega}) \quad = \quad \frac{1}{n-1} \sum_{j \neq i} w_{ij}(\boldsymbol{\Omega}) \qquad \forall i \in N$$

We have assumed an unweighted, undirected network so that ingoing and outgoing measures are symmetric, but alternatives are easily computed. There are several related measures of centrality, in particular diffusion centrality and communication centrality (Banerjee et al., 2013, 2014), Bonacich centrality (Bonacich, 1987), information centrality (Stephenson and Zelen, 1989), random walk closeness centrality (Noh and Rieger, 2004), cascade centrality (Teytelboym et al., 2015), and percolation centrality (Moore and Newman, 2000; Piraveenan et al., 2013) in the epidemiological literature. As far as we are aware, no measure uses probabilities of information travelling by diffusion between two nodes.

A.1. Comparison with diffusion centrality.

Of particular interest is the relationship between our measure and diffusion centrality. Banerjee et al. (2013) empirically investigate the effects of information in social networks on the decisions of individuals to take up a microfinance opportunity in villages in India. They develop diffusion centrality as an approximation of *communication centrality*, a simulated measure linked to the Susceptible, Infected, Recovered model (Kermack and McKendrick, 1927; Bailey, 1957), which runs for a finite period of time and allows for non-participants to pass on the message. Diffusion centrality has a 0.86 correlation with communication centrality. It is given by $d_i = \left[\sum_{\tau=1}^{T} (p\mathbf{G})^{\tau}\mathbf{1}\right]_i$ where p is equal to the inverse of the largest eigenvalue of the adjacency matrix, $\lambda_{max}(\mathbf{G})$. Banerjee et al. (2013) choose this critical value of $1/\lambda_{max}(\mathbf{G})$ because *"the entries of $[p\boldsymbol{G}]^T$ tend to 0 as T grows if $p < 1/\lambda_{max}(\boldsymbol{G})$, and some entries diverge if $p > 1/\lambda_{max}(\boldsymbol{G})$"*. As T tends to infinity, diffusion centrality becomes proportional to either Bonacich centrality or eigenvector centrality, depending on whether p is smaller or larger than $1/\lambda_{max}(\mathbf{G})$, respectively.

FIGURE 7. Comparing centrality measures in a village.
Link-level probability $p = 1/\lambda_{max}(\mathbf{G}) = 0.06$ (L), and $p = 0.15$ (R)



As we showed in Section 4, diffusion centrality does not take account of double counting, measuring instead the total amount of information travelling between nodes in a network. This means it overemphasises the benefit of hearing a lot of information, because at some point extra information is redundant if these signals are likely to have already been received via other walks in the network. Diffusion centrality has been found to have empirical relevance (Breza and Chandrasekhar, 2015; Fafchamps and Labonne, 2016), and our word-of-mouth probabilities may be easier to work with in an empirical context for two reasons. First, they lie between zero and one and so no transformation is required to approximate a probability, as is the case for (Breza and Chandrasekhar, 2015). Secondly, they can be calculated for any value of the probability that two neighbours talk, not just when $p = 1/\lambda_{max}(\mathbf{G})$.

Figure 7 compares diffusion centrality and word-of-mouth centrality, using data from one of the Indian villages studied by Banerjee et al. (2013). Centralities are calculated at the household level.[30] The value of $p = 1/\lambda_{max}(\mathbf{G})$ is used to compare the two centralities in the left chart, and there is clearly a strong relationship between the two measures. In fact, at this value of p, a linear transformation of diffusion centrality would be a good approximation of the node-to-node probabilities, and this approach is used by Breza and Chandrasekhar (2015). The chart suggests a transformation factor of 0.06.

The chart on the right shows the comparison between the two measures when we use a larger p, and while ranking seems to be generally preserved, there is some divergence in the relative magnitude of the measures, as the word-of-mouth centralities converge towards 1. So if there is any reason to suspect that p differs from $1/\lambda_{max}(\mathbf{G})$, word-of-mouth centrality may be useful for calculating probabilities of information transmission by diffusion in a network. In particular, we can observe that the level of inequality in diffusion centrality between the nodes in a network is higher than when word-of-mouth centrality is used. This is because central nodes who receive a lot of information have extremely high values of diffusion centrality. But with word-of-mouth centrality, the effect of this extra information is discounted due to the fact that it is probably redundant. Central nodes have most likely received the signal via other walks already. This analysis suggests that diffusion centrality overemphasises the benefit of a central network position in relation to information transmission.

A.2. **Ranking of nodes.** We can also find an example network, shown in Figure 8, where the ranking of the centralities differs between diffusion and word-of-mouth, and in fact changes with the parameters. Figure 9 shows how the word-of-mouth centralities for this network change as T increases (with $p = 0.34$ — just slightly above $1/\lambda_{max}(\mathbf{G}) = 0.32$). We can observe that:

[30]Village number 1; $n = 182$. For this comparison we use $w_i = \frac{1}{n}\sum_{j\in N} w_{ij}$ (i.e. not excluding w_{ii} compared to the formula in Definition A.1). This makes it more comparable with diffusion centrality, which includes the diagonal entries of the matrix in its sum.

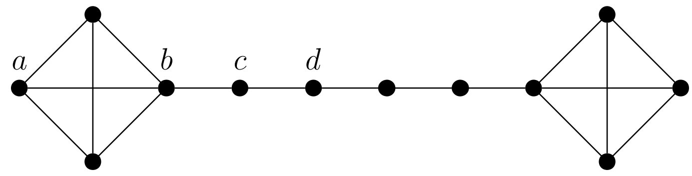
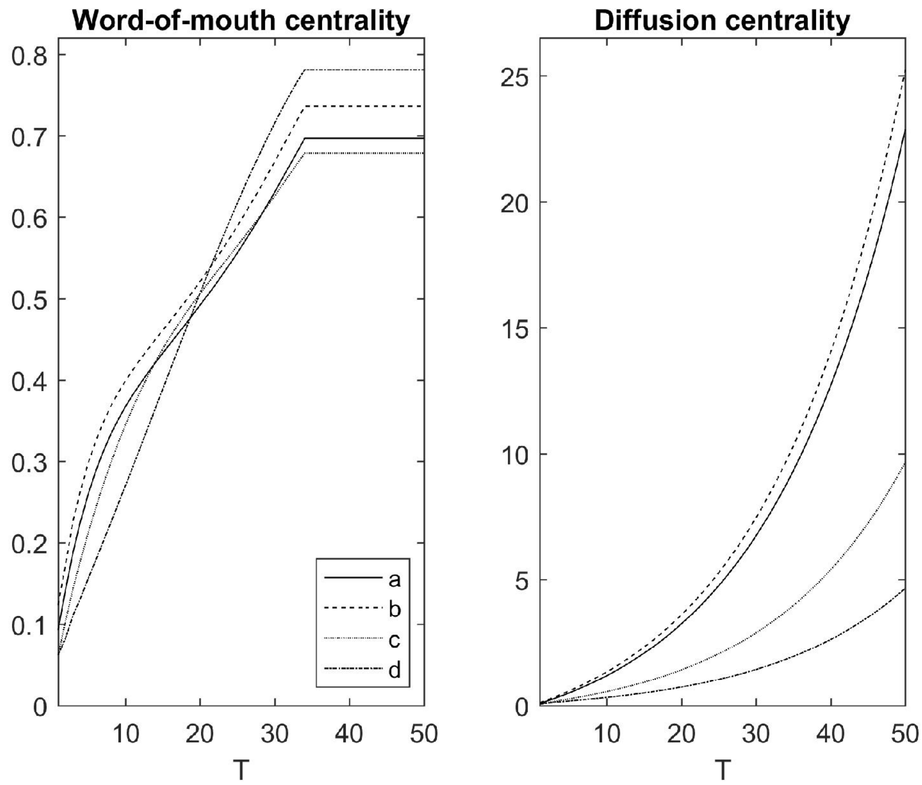


FIGURE 8. A network of 12 nodes, 4 positions

FIGURE 9. Comparing centrality measures as T increases



- Word-of-mouth centrality converges to an upper bound which is less than 1, whereas diffusion centrality grows beyond that
- The relative rankings of the two centralities are the same when T is low (less than 5), but after this point the rankings in word-of-mouth centrality switch around.
- This switch may be due to the fact that, at low T, short walks are more important, and node positions a and b are most central. When T is higher, longer walks are also feasible and now nodes d overtakes a and c to have the second highest centrality

- This change is not observable with diffusion centrality, perhaps because the summation formula does not allow for these differences to have an impact.
- With word-of-mouth centrality, rankings depend on the parameters p and T, and these parameters could be empirically observable.

Appendix B. Proofs

B.1. **Proof of Proposition 4.2.** Obstructed word-of-mouth probabilities and the information structure $\mathbf{\Omega}$.

(1) **Increasing** p: We have that $\frac{\partial w_{ij}(p,T,\mathbf{G})}{\partial p} \geq 0$ with a strict inequality if and only if $l_{ij}(\tau, \mathbf{G}) > 0$ for any $\tau \leq T$, that is, if and only if i and j are connected by one or more walks of length $\leq T$ in the network $\mathbf{G}$.

(2) **Increasing** T **to** $T+1$: Let

$$F_{ij}(p, T, \mathbf{G}) = 1 - w_{ij}(\mathbf{\Omega}) = \prod_{\tau=1}^{T} [1 - p^{\tau}]^{l_{ij}(\tau, \mathbf{G})} \tag{B.1}$$

If T increases to $T+1$, we have that

$$F_{ij}(p, T+1, \mathbf{G}) = F_{ij}(p, T, \mathbf{G})(1 - p^{(T+1)})^{l_{ij}(T+1, \mathbf{G})} \tag{B.2}$$

So $F_{ij}(p, T+1, \mathbf{G}) < F_{ij}(p, T, \mathbf{G})$ if and only if $l_{ij}(T+1, \mathbf{G}) \geq 1$; otherwise they are equal. The same argument holds for any further increases in T. Hence $F_{ij}(\mathbf{\Omega})$ is weakly monotonically decreasing in T and so $w_{ij}(\mathbf{\Omega})$ is weakly monotonically increasing in T.

(3) **Adding a link to the information network**: Suppose that we add a link to the network $\mathbf{G}$, creating the network $\mathbf{G}'$, which has an additional walk of length τ between i and j so that $l_{ij}(\tau, \mathbf{G}') = l_{ij}(\tau, \mathbf{G}) + 1$. Now we want to compare $F_{ij}(p, T, \mathbf{G}')$ and $F_{ij}(p, T, \mathbf{G})$. Since

$$F_{ij}(p, T, \mathbf{G}') = F_{ij}(p, T, \mathbf{G})(1 - p^{\tau}) \tag{B.3}$$

We have that $F_{ij}(p, T, \mathbf{G}') < F_{ij}(p, T, \mathbf{G})$ as required. Meanwhile any change to the network that leaves $l_{ij}(\tau)$ unchanged $\forall \tau \leq T$ leaves the probabilities unchanged.

B.2. **Proof of Proposition 6.1.** By Remark 6.1, the network is informative if and only if $\mathbf{G}$ is connected and $D_{\mathbf{G}} \leq T$, and player k is visible if and only if $\mathbf{G}_{-k}$ is connected and $D_{\mathbf{G}_{-k}} \leq T$. Since removing a node from the network cannot connect a disconnected network, and can only increase social distances so that $d_{ij}(\mathbf{G}_{-k}) \geq d_{ij}(\mathbf{G}) \;\; \forall\, i, j, k \in N$, if $\mathbf{G}_{-k}$ is connected and $D_{\mathbf{G}_{-k}} \leq T$, this implies that $\mathbf{G}$ is connected and $D_{\mathbf{G}} \leq T$: i.e. a player's visibility implies the original network $\mathbf{G}$ is informative. When the network is informative, $w_{ij}(\mathbf{G}) > 0 \; \forall \; i, j \in N$, and so k is visible if and only if he is not obstructive.

B.3. **Proof of Remark 3.2.** $Q_i^j(k)$ is the conditional probability that, if player k is a B-type, an S-type player i who matched with another S-type player j in period 1 will hear a signal about player k:

$$Q_i^j(k) \quad \equiv \quad \Pr\left[\rho_i(k) = 1 \mid \{k \text{ is B-type}\}, \;\; r^i = r^j = 0, \;\; k, \mu_k^1 \notin \{i, j\}\right]$$

By the law of total expectations, $Q_i^j(k)$ is given by:

$$\begin{aligned}&\Pr\left[\rho_i(k) = 1 \mid \{k \text{ is B-type}\}, \;\; r^i = r^j = 0, \;\; k, \mu_k^1 \notin \{i, j\}\right] \\ &= \sum_{h \in N} \left[\Pr\left[\rho_i(k) = 1 \mid \{k \text{ is B-type}\}, \; r^i = r^j = 0, \;\; k, \mu_k^1 \notin \{i, j\}, \; \mu_k^1 = h\right] \Pr\left[\mu_k^1 = h \mid k, \mu_k^1 \notin \{i, j\}\right]\right]\end{aligned}$$

The second term is

$$\begin{aligned}\Pr\left[\mu_k^1 = h \mid k, \mu_k^1 \notin \{i, j\}\right] \quad &= \quad \frac{\Pr\left[(\mu_k^1 = h) \cap (k, \mu_k^1 \notin \{i, j\})\right]}{\Pr\left[k, \mu_k^1 \notin \{i, j\}\right]} \\ &= \quad \begin{cases} \frac{1/(n-1)}{\sum_{h \neq i,j} 1/(n-1)} = \frac{1}{n-2} & \forall \; h \notin \{i, j\} \\ 0 & \forall \; h \in \{i, j\} \end{cases}\end{aligned}$$

The first term is

$$\Pr\left[\rho_i(k) = 1 \mid \{k \text{ is B-type}\}, \; r^i = r^j = 0, \; k, \mu_k^1 \notin \{i, j\}, \; \mu_k^1 = h\right] = \begin{cases} 0 & \forall \; k \in \{i, j\} \\ 0 & \forall \; h \in \{i, j\} \\ p_{hi}(k) & \text{otherwise} \end{cases}$$

Therefore $Q_i^j(i) = Q_i^j(j) = 0$, and otherwise we have that:

$$Q_i^j(k) = \frac{1}{n-2} \sum_{h \neq i,j} p_{hi}(k) \qquad \forall \; k \notin \{i, j\} \tag{B.4}$$

B.4. **Proof of Proposition 4.1.** In Definition 4.1 we assumed that the probability of information travelling along each walk is independent, implying that players only recall signals they receive in the last round of information transmission, and forget signals received and passed on in earlier rounds. There are four cases.

An S-type player and a signal about himself: This S-type player has deviated in period 1. He will have strictly lower payoffs if his future partner is an S-type and has heard about his deviation, because they will exit against him instead of defecting. His period 3 payoffs are unchanged if he meets a B-type (because a B-type would not 'punish him'. He could match with anyone in period 3 (because $m_{ij} > 0 \ \forall \ j \neq i$) and because he does not know which of these players are S-types and which are B-types, he does not want any of them to find out about his deviation. Therefore he will strictly prefer to conceal a signal about himself from all other players. This holds for all rounds of information transmission.

An S-type player and a signal about someone else: S-type players are not expecting another S-type to deviate, so a signal about another player would tell them that he is a B-type. An S-type player who has heard a signal about a B-type player, and meets him in period 3, has payoff 0. If he has not heard the signal, his payoff on meeting a B-type is $-\beta$. If he meets an S-type, his payoffs are unchanged by hearing a signal about a B-type. As he could meet any of the players in period 3, an S-type player strictly prefers to receive a signal about a B-type.

However it is not the case that passing on a signal always increases the probability that it is received. Sometimes it may have no effect. For example, a player passing on a signal in round T cannot increase his probability of receiving a signal, since it will not have time to return to him, as there are no more rounds of information transmission. So he is indifferent between the actions of either passing on a signal or not in round T. However, if players do not pass on the signal in round T, they cannot increase their probability of receiving it by passing it on in round $T-1$, and so information transmission could quickly unravel. Specifically, a player can strictly increase his probability of receiving a signal by passing it on, if and only if other players pass it on in the following rounds, and he is part of a walk that returns to his network position in a number of links which is a factor of the number of information transmission rounds remaining. We can observe that the strategy for *all* S-type players to pass on signals about other players in *all* rounds is weakly preferred.

A B-type player and a signal about himself: A B-type's payoffs are $x/2$ if his future partner has heard the signal and x otherwise, so he strictly prefers to conceal a signal about himself.

A B-type player and a signal about someone else: In this case, a B-type's payoffs in period 3 are not affected by whether he has heard a signal about another B-type or not. So he is indifferent between passing signals about other players and not passing them. Therefore the strategy to pass on signals about all other players in all rounds is weakly preferred. This means that in our model a B-type player has the same strategy for passing on signals as does an S-type. This seems reasonable: a B-type player may not wish to draw attention to himself by not passing on signals about other players, when it would not decrease his payoffs to do so.

Secondly, in our equilibrium the victim of a deviation truthfully reports it, because he is indifferent and so weakly prefers to tell the truth. On the other hand, a player does not have incentives to fabricate a report of a deviation, because if he hears his own false report through the network and believes it, this could lead him to exit against an S-type player, and hence miss out on positive payoffs. So in our model, players do not have incentives to lie.

B.5. **Proof of Proposition 4.3.** We want to show that period 3 payoffs are increasing in obstructed centrality given in Definition 4.4. Ex ante, i could meet any player k in period 3, and any player j in period 1. So we want to find V_i, the average expected period 3 payoffs over any k and any j, and check that it is (weakly) increasing in P_i.

$$\begin{aligned} V_i &= \frac{1}{(n-1)^2}\sum_{k\neq i}\sum_{j\neq i} V_i^j(k) \\ &= \frac{1}{(n-1)^2}\sum_{k\neq i}\sum_{j\neq i}\left[\sigma(1-\phi)-\beta\phi(1-Q_i^j(k))\right] \\ &= \sigma(1-\phi)-\beta\phi+\beta\phi\frac{1}{(n-1)^2}\sum_{k\neq i}\sum_{j\neq i} Q_i^j(k) \end{aligned} \tag{B.5}$$

Let $P_i(k) = \frac{1}{n-1}\sum_{h\neq i} p_{hi}(k)$, noting that $\frac{1}{n-1}\sum_{k\neq i} P_i(k) = P_i$. We can substitute this into the expression for $Q_i^j(k)$ (B.4) to give

$$Q_i^j(k) = \frac{1}{n-2}\left[\sum_h p_{hi}(l) - p_{ji}(k)\right] = \frac{1}{n-2}\left[(n-1)P_i(k) - p_{ji}(k)\right]$$

Next we take the averages required

$$\begin{aligned} \frac{1}{(n-1)^2}\sum_{k\neq i}\sum_{j\neq i} Q_i^j(k) &= \frac{1}{(n-1)^2}\sum_{k\neq i}\sum_{j\neq i}\frac{1}{n-2}\left[(n-1)P_i(k)-p_{ji}(k)\right] \\ &= \frac{1}{(n-1)(n-2)}\sum_{k\neq i}\left[(n-1)P_i(k)-\frac{1}{(n-1)}\sum_{j\neq i}p_{ji}(k)\right] \\ &= \frac{1}{(n-2)}\left[(n-1)P_i-\frac{1}{(n-1)^2}\sum_{k\neq i}\sum_{j\neq i}p_{ji}(k)\right] \\ &= \frac{1}{(n-2)}\left[(n-1)P_i - P_i\right] = P_i \end{aligned}$$

And therefore from (B.5) we have that $\frac{\partial V_i}{\partial P_i} = \beta\phi > 0$, as required.

B.6. **Proof of Proposition 5.4.** The expression for each player's utility given in Proposition 5.3 is strictly increasing in both the number of cooperative partnerships, and in period 3 payoffs. From Proposition 5.2 the number of cooperative partnerships is weakly increasing in the losses in each partnership, and from (5.4) losses are strictly increasing in information transmission probabilities. From Proposition 4.3, period 3 payoffs are increasing in information transmission probabilities. Putting these together, overall utility is weakly increasing in the probabilities of information transmission. So more information, in the sense of greater probabilities of information transmission, weakly increases the levels of cooperation and welfare in this repeated game. Finally, Proposition 4.2 showed that the probabilities of information transmission are weakly increasing in the three aspects of the information structure $\mathbf{\Omega} = p, T, \mathbf{G}$.

B.7. **Proof of Remark 6.3.** There are $\frac{n!}{2(n-2)!}$ pairs in a network of n nodes, and if L is the number of visible nodes, there are $\frac{L!}{2(L-2)!}$ pairs where both partners are visible. In an informative tree network, all nodes except the leaf nodes are obstructive, because leaf nodes are the only ones who would not disconnect the network by their absence. Nodes are defined as visible if they are not obstructive in an informative network, so only the leaf nodes in a tree network are visible. All nodes except the centre of the star are leaf nodes, and so the star has the maximum number of visible nodes for any tree: $L = n - 1$.

Appendix C. Additional results

C.1. **A good reputation.** In our framework, an alternative specification with a good signal, that was emitted when one's partner cooperated, would work as follows.

Period 3: For punishment to work, there would be a parametric assumption that a player would only be rewarded for cooperation in the first round if a good signal about them was received by their final-round partner. This means that if a player received a good signal about their partner in the final round they would they defect, and otherwise they would punish (exit). Hence Assumption 1 could no longer hold. And it would imply that some S-type players would be punished in equilibrium, because a signal was not received by their partner - in contrast to our model with a bad signal, where only the B-types are punished.

Period 2: With a good signal, this would mean everyone had an incentive to pass on a signal about themselves and about other players. This is because the reward for cooperation (mutual defection) would only be possible if both partners received a good signal about each other. This means there would be no obstruction. However, there would be an incentive to fabricate false reports of one's own cooperation (whether or not they were true), to increase the likelihood that such a report would be received by one's future partner.

Period 1: In the first round, incentives to be honest would depend on the probability that a good signal emitted by a player's partner would reach his future partners, in the same way as the current model - but without obstruction. However, because Assumption 1 would not hold, players who could not cooperate would coordinate on the Nash equilibrium of mutual exit, rather than mutual defection.

This is an interesting thought experiment because it suggests that there may be different welfare impacts of either good or bad signals in different networks and with different parameters. On one hand, cooperation might be higher with a good signal because there is no obstruction and so greater probabilities of information transmission. But on the other hand, welfare might be lower because some S-types would be wrongly punished in period 3, and because non-cooperating partners in period 1 could not defect, only exit.

C.2. **Obstruction by subsets of nodes in different rounds of information transmission.** Let X_τ be the subset of nodes who obstruct a signal in round τ of information transmission. Let $\mathbf{X} = \{X_\tau \subset N,\, 1 \leq \tau \leq T\}$ be the set of those subsets. Let $l_{ij}(\tau, \mathbf{G}, \mathbf{X})$ be the number of walks between i and j of length τ when the set of obstructing nodes is $\mathbf{X}$.

To calculate this, recall that nodes only remember the information they receive in the last round. So longer walks will not connect to other nodes, if the links it would traverse are those which connect a node who is obstructing in the relevant round of information transmission. So we have that, for example, $l_{ij}(1, \mathbf{G}, \mathbf{X}) = [\mathbf{G}_{-X_1}]_{ij}$ where $\mathbf{G}_{-X_1}$ is the network $\mathbf{G}$ with those nodes in X_1 removed. Then we have that $l_{ij}(2, \mathbf{G}, \mathbf{X}) = [\mathbf{G}_{-X_2}\mathbf{G}_{-X_1}]_{ij}$ and $l_{ij}(3, \mathbf{G}, \mathbf{X}) = [\mathbf{G}_{-X_3}\mathbf{G}_{-X_2}\mathbf{G}_{-X_1}]_{ij}$, and so on. In general, $l_{ij}(\tau, \mathbf{G}, \mathbf{X}) = [\prod_{\tau=1}^{\mathcal{T}} \mathbf{G}_{-X_\tau}]_{ij}$, ensuring that the ordering of matrices $\mathbf{G}$ is preserved.

C.3. **Updated subjective probabilities.** If i meets k in period 3 and has not heard any signal, there are two possibilities: either k is an S-type; or k is a B-type but i has not heard about it. Player i will still defect against the unknown player k due to Assumption 1. Now $\phi_i^j(k)$ is his updated belief that k is a B-type player, given that he has heard no signal about him, that is:

$$\phi_i^j(k) \equiv \Pr[\{k \text{ is B-type}\} \mid \rho_i(k) = \emptyset, \quad r^i = r^j = 0, \quad k, \mu_k^1 \notin \{i, j\}]$$

Let $Q_i^j(k)$ be the conditional probability that, if k is a B-type, an S-type player i who matched with another S-type player j in period 1 will hear a signal about player k (see next subsection).

$$Q_i^j(k) \equiv \Pr\left[\rho_i(k) = 1 \mid \{k \text{ is B-type}\}, \quad r^i = r^j = 0, \quad k, \mu_k^1 \notin \{i, j\}\right]$$

To find $\phi_i^j(k)$, we need the following expressions:

$$\begin{aligned}
\Pr\left[\rho_i(k) = \emptyset \mid \{k \text{ is S-type}\}, \quad r^i = r^j = 0, \quad k, \mu_k^1 \notin \{i, j\}\right] &= 1 \\
\Pr\left[\rho_i(k) = \emptyset \cap \{k \text{ is S-type}\} \mid r^i = r^j = 0, \quad k, \mu_k^1 \notin \{i, j\}\right] &= 1 - \phi \\
\Pr\left[\rho_i(k) = \emptyset \mid \{k \text{ is B-type}\}, \quad r^i = r^j = 0, \quad k, \mu_k^1 \notin \{i, j\}\right] &= 1 - Q_i^j(k) \\
\Pr\left[\rho_i(k) = \emptyset \cap \{k \text{ is B-type}\} \mid r^i = r^j = 0, \quad k, \mu_k^1 \notin \{i, j\}\right] &= \phi(1 - Q_i^j(k))
\end{aligned}$$

Using the final equation from this list and Bayes' rule we have that

$$\begin{aligned}
\phi_i^j(k) &= \Pr[\{k \text{ is B-type}\} \mid \rho_i(k) = \emptyset, \quad r^i = r^j = 0, \quad k, \mu_k^1 \notin \{i, j\}] \\
&= \frac{\phi(1 - Q_i^j(k))}{\phi(1 - Q_i^j(k)) + 1 - \phi} = \frac{\phi(1 - Q_i^j(k))}{1 - \phi Q_i^j(k)} \qquad \text{(C.1)}
\end{aligned}$$

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School of Economics and Finance

Queen Mary
University of London

This working paper has been produced by the School of Economics and Finance at Queen Mary University of London

School of Economics and Finance
Queen Mary University of London
Mile End Road
London E1 4NS
Tel: +44 (0)20 7882 7356
Fax: +44 (0)20 8983 3580
Web: www.econ.qmul.ac.uk/research/workingpapers/